Math 290B Friday, March 27 Problem Session

- Mon B RQ 6AM
- Tue D
- Wed Exam M (usual time)
- Thu PD
- Fri Problems, Writing (?)

Office Hours by appointment

VS. M20

$p(x)=8$

$p(x)=0$

P_n = all polys w/ degree less than or equal to n.

E_n = all polys w/ degree exactly n. Subspace?

E_3 $p = 2 + x + 5x^2 - 6x^3$ $q = 8 + 6x^3$ $p, q \in E_3$

Compute $p + q = 10 + x + 5x^2 \notin E_3$

($x + 5x^2$: degree 2)

So additive closure does not hold. Violates Defn VS, E_3 not a vector space, ~~so not a subspace of~~

S. C25 $W = \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \middle| \, 3x_1 - 5x_2 = 12 \right\} \subseteq \mathbb{C}^2$

AC $\underset{\sim}{x}, \underset{\sim}{y} \in W \Rightarrow \underset{\sim}{x} + \underset{\sim}{y} \in W$ for all $\underset{\sim}{x}, \underset{\sim}{y}$

Property AC $\underset{\sim}{x} = \begin{bmatrix} 4 \\ 0 \end{bmatrix} \in W \quad \underset{\sim}{y} = \begin{bmatrix} -1 \\ -3 \end{bmatrix} \in W$

$\underset{\sim}{x} + \underset{\sim}{y} = \begin{bmatrix} 3 \\ -3 \end{bmatrix} \quad 3(3) - 5(-3) = 24 \neq 12$ so $\underset{\sim}{x} + \underset{\sim}{y} \in W$

"counter example"

LISS. T40 A, B $m \times n$ matrices, $C = \{ \underset{\sim}{u}_1, \underset{\sim}{u}_2, \ldots \underset{\sim}{u}_p \} \subseteq \mathbb{C}^n$ lin ind, spans $\mathbb{C}^n$.

(See Section A transcript.)

$A\underset{\sim}{u}_i = B\underset{\sim}{u}_i$ for $1 \leq i \leq n \Rightarrow A = B$

Proof $\underset{\sim}{e}_i$ = column i of identity matrix I_n

$A = [\underset{\sim}{A}_1 | \underset{\sim}{A}_2 | \cdots | \underset{\sim}{A}_n] \quad B = [\underset{\sim}{B}_1 | \underset{\sim}{B}_2 | \cdots | \underset{\sim}{B}_n]$

$\underset{\sim}{A}_i = [\underset{\sim}{A}_1 | \underset{\sim}{A}_2 | \cdots | \underset{\sim}{A}_n] \, \underset{\sim}{e}_i = A\underset{\sim}{e}_i$

$$= A(a_1\underset{\sim}{u}_1 + a_2\underset{\sim}{u}_2 + \cdots + a_p\underset{\sim}{u}_p)$$ C spans $\mathbb{C}^n$

$$= a_1A\underset{\sim}{u}_1 + a_2A\underset{\sim}{u}_2 + \cdots + a_pA\underset{\sim}{u}_p$$ MMDAA

$$= a_1B\underset{\sim}{u}_1 + a_2B\underset{\sim}{u}_2 + \cdots + a_pB\underset{\sim}{u}_p$$ Hypothesis

$$= B(a_1\underset{\sim}{u}_1 + a_2\underset{\sim}{u}_2 + \cdots + a_p\underset{\sim}{u}_p)$$ MMDAA

$$= B\underset{\sim}{e}_i$$

$$= [\underset{\sim}{B}_1 | \underset{\sim}{B}_2 | \cdots | \underset{\sim}{B}_n]\underset{\sim}{e}_i$$

$$= \underset{\sim}{B}_i \quad \text{for } 1 \le i \le n$$

So A & B have identical columns, so $A = B$.