

Math 290 B, Thursday, April 9 Section EE

Thu - EE (Sage)

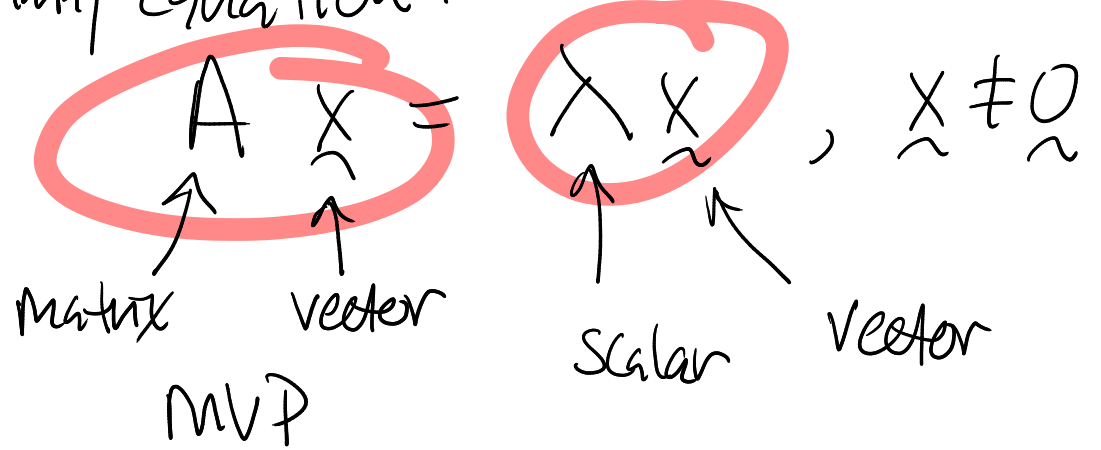
Fri - PEE

Mon - SD (Sage)

Tue - Problems Writing D&E

Wed - Exam D&E

Defining equation:



MVP

Diagonalization

$$A \underline{x} = \lambda \underline{x} \quad \underline{x} \neq \underline{0}$$

$$A \underline{x} - \lambda \underline{x} = \underline{0}$$

$$A \underline{x} - \lambda I \underline{x} = \underline{0}$$

$$(A - \lambda I) \underline{x} = \underline{0} \quad \lambda \neq 0$$

$A - \lambda I$ singular

$$\det(A - \lambda I) = 0 \quad \text{SMEZ}$$

characteristic polynomial

$$\underline{x} \in \underbrace{N(A - \lambda I)}_{\text{eigen space}} = \mathcal{E}_A(\lambda)$$

Ex $A = \begin{bmatrix} 8 & -6 & 6 \\ 6 & -4 & 6 \\ -3 & 3 & -1 \end{bmatrix}$ Eigen-stuff?

$$P_A(\lambda) = \det(A - \lambda I) = \begin{vmatrix} 8-\lambda & -6 & 6 \\ 6 & -4-\lambda & 6 \\ -3 & 3 & -1-\lambda \end{vmatrix} = -\lambda^3 + 3\lambda^2 - 4\lambda = -(\lambda-2)^2(\lambda+1)$$

Eigenvalues $\lambda=2$ $\alpha_A(2)=2$
 $\lambda=-1$ $\alpha_A(-1)=1$

Eigenspaces

$\lambda=2$ $A - 2I_3 = \begin{bmatrix} 6 & -6 & 6 \\ 6 & -6 & 6 \\ -3 & 3 & -3 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$e_A(2) = N(A - 2I_3) =$

$\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$

$\alpha_A(2)=2$
 $\dim = 2$

$$\lambda = -1 \quad A - (-1)I_3 = A + I_3 = \begin{bmatrix} 9 & -6 & 6 \\ 6 & -3 & 6 \\ -3 & 3 & 0 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 0 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathcal{E}_A(-1) = N(A + I_3) = \left\langle \begin{bmatrix} -2 \\ -2 \\ 1 \end{bmatrix} \right\rangle \quad \chi_A(-1) = 1$$

$\dim = 1$