Math 290 B, Thursday, April 9 Section EE

Thu - EE (Sage)

Fri - PEE

Mon - SD (Sage)

Tue - Problems Writing D&E

Wed - Exam D&E

Defining equation:

$$\underbrace{A}_{\text{matrix}}\ \underbrace{\underset{\sim}{x}}_{\text{vector}} = \underbrace{\lambda}_{\text{scalar}}\ \underbrace{\underset{\sim}{x}}_{\text{vector}}, \quad \underset{\sim}{x} \neq \underset{\sim}{0}$$

MVP

Diagonalization

$$A\underset{\sim}{x} = \lambda \underset{\sim}{x} \qquad \underset{\sim}{x} \neq \underset{\sim}{0}$$

$$A\underset{\sim}{x} - \lambda \underset{\sim}{x} = \underset{\sim}{0}$$

$$A\underset{\sim}{x} - \lambda I \underset{\sim}{x} = \underset{\sim}{0}$$

$$(A - \lambda I)\underset{\sim}{x} = \underset{\sim}{0} \qquad \underset{\sim}{x} \neq \underset{\sim}{0}$$

$A - \lambda I$ singular

$\underbrace{\det(A - \lambda I)}_{\text{characteristic polynomial}} = 0$ SMED

$\underset{\sim}{x} \in \underbrace{N(A - \lambda I)}_{\text{eigen space}} = \mathcal{E}_A(\lambda)$

Ex $A = \begin{bmatrix} 8 & -6 & 6 \\ 6 & -4 & 6 \\ -3 & 3 & -1 \end{bmatrix}$ eigen-stuff?

$$p_A(\lambda) = \det(A - \lambda I) = \begin{vmatrix} 8-\lambda & -6 & 6 \\ 6 & -4-\lambda & 6 \\ -3 & 3 & -1-\lambda \end{vmatrix} = -\lambda^3 + 3\lambda^2 - 4$$
$$= -(\lambda-2)^2(\lambda+1)$$

Eigenvalues $\lambda = 2$ $\quad \alpha_A(2) = 2$

$\lambda = -1$ $\quad \alpha_A(-1) = 1$

Eigenspaces

$\lambda = 2$ $\quad A - 2I_3 = \begin{bmatrix} 6 & -6 & 6 \\ 6 & -6 & 6 \\ -3 & 3 & -3 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$$\mathcal{E}_A(2) = N(A - 2I_3) = \left\langle \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\} \right\rangle$$

$\gamma_A(2) = 2$

dim $= 2$

$\lambda = -1 \qquad A-(-1)I_3 = A + I_3 = \begin{bmatrix} 9 & -6 & 6 \\ 6 & -3 & 6 \\ -3 & 3 & 0 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

$\mathcal{E}_A(-1) = N(A+I_3) = \left\langle \left\{ \begin{bmatrix} -2 \\ -2 \\ 1 \end{bmatrix} \right\} \right\rangle \qquad \gamma_A(-1) = 1$

dim $= 1$