Math 290 B, Friday, April 10, Section PEE

Mon- SD (Sage)

Tue- Problem Session
Writing D&E
Bring Your Pet to Class Day

Wed - Exam D&E

Office Hours
Same times (Pacific)
Google Meet

Theorem EDELI $S = \{\underset{\sim}{x}_1, \underset{\sim}{x}_2, \dots, \underset{\sim}{x}_p\}$ eigenvectors of A for eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_p$ with $\lambda_i \neq \lambda_j$. Then S is linearly independent

<u>Proof</u> By contradiction; Assume S is linearly dependent.

There is an index k so that $\{\underset{\sim}{x}_1, \underset{\sim}{x}_2, \dots, \underset{\sim}{x}_{k-1}\}$ linearly independent

$\{\underset{\sim}{x}_1, \underset{\sim}{x}_2, \dots, \underset{\sim}{x}_{k-1}, \underset{\sim}{x}_k\}$ linearly dependent set

There are scalars, not all zero, so that

$$\underset{\sim}{0} = a_1\underset{\sim}{x}_1 + a_2\underset{\sim}{x}_2 + \dots + a_k\underset{\sim}{x}_k$$

① $\underset{\sim}{0} = \lambda_k \underset{\sim}{0} = \lambda_k(a_1\underset{\sim}{x}_1 + a_2\underset{\sim}{x}_2 + \dots + a_k\underset{\sim}{x}_k) = a_1\lambda_k\underset{\sim}{x}_1 + a_2\lambda_k\underset{\sim}{x}_2 + \dots \boxed{+ a_k\lambda_k\underset{\sim}{x}_k}$

② $\underset{\sim}{0} = A\underset{\sim}{0} = A(a_1\underset{\sim}{x}_1 + a_2\underset{\sim}{x}_2 + \dots + a_k\underset{\sim}{x}_k)$

$= a_1A\underset{\sim}{x}_1 + a_2A\underset{\sim}{x}_2 + \dots + a_kA\underset{\sim}{x}_k$

$= a_1\lambda_1\underset{\sim}{x}_1 + a_2\lambda_2\underset{\sim}{x}_2 + \dots \boxed{+ a_k\lambda_k\underset{\sim}{x}_k}$

MMDAA

Subtract ① from ②

$$\underset{\sim}{0} - \underset{\sim}{0} = a_1\lambda_1\underset{\sim}{x}_1 - a_1\lambda_k\underset{\sim}{x}_1 + a_2\lambda_2\underset{\sim}{x}_2 - a_2\lambda_k\underset{\sim}{x}_2 + \dots + a_{k-1}\lambda_{k-1}\underset{\sim}{x}_{k-1} - a_{k-1}\lambda_k\underset{\sim}{x}_{k-1}$$

$$\underset{\sim}{0} = a_1(\lambda_1-\lambda_k)\underset{\sim}{x}_1 + a_2(\lambda_2-\lambda_k)\underset{\sim}{x}_2 + \dots + a_{k-1}(\lambda_{k-1}-\lambda_k)\underset{\sim}{x}_{k-1}$$

This is a RLD on a LI set!

So $a_1\underbrace{(\lambda_1-\lambda_k)}_{\text{nonzero}}=0,\quad a_2\underbrace{(\lambda_2-\lambda_k)}_{\text{non zero}}=0,\quad \dots,\quad a_{k-1}\underbrace{(\lambda_{k-1}-\lambda_k)}_{\text{nonzero}}=0$

So $a_1=0,\ a_2=0,\ \dots,\ a_{k-1}=0$

So $a_k \neq 0$ so $a_k\underset{\sim}{x}_k=\underset{\sim}{0} \Rightarrow \overset{\text{eigenvector}}{\underset{\sim}{x}_k}=\underset{\sim}{0} \cancel{\Rightarrow}$

Theorem HMRE A Hermitian $\Rightarrow$ all eigenvalues real

<u>Proof</u> Let λ be an eigenvalue of A with eigenvector $\underset{\sim}{x}$.

$$\lambda\langle \underset{\sim}{x}, \underset{\sim}{x}\rangle = \langle \underset{\sim}{x}, \lambda \underset{\sim}{x}\rangle = \langle \underset{\sim}{x}, A\underset{\sim}{x}\rangle = \langle A\underset{\sim}{x}, \underset{\sim}{x}\rangle = \langle \lambda\underset{\sim}{x}, \underset{\sim}{x}\rangle = \bar{\lambda}\langle \underset{\sim}{x}, \underset{\sim}{x}\rangle$$

A Hermitian

$\langle \underset{\sim}{x}, \underset{\sim}{x}\rangle \neq 0$ "Cancel" $\Rightarrow \lambda = \bar{\lambda} \Rightarrow \lambda \in \mathbb{R}$

Theorem PIP, $\underset{\sim}{x} \neq \underset{\sim}{0}$ $\langle \underset{\sim}{x}, \underset{\sim}{x}\rangle$