Math 290B, Thursday, April 16, Section LT

$T: \underset{\sim}{U} \to V$ $\quad$ T functions, $\underset{\sim}{U}$ & V vector spaces

1) $T(\underset{\sim}{u}_1 + \underset{\sim}{u}_2) = \underbrace{T(\underset{\sim}{u}_1)} + \underbrace{T(\underset{\sim}{u}_2)}$ $\quad$ element of V

2) $T(\alpha \underset{\sim}{u}) = \alpha \underbrace{T(\underset{\sim}{u})}$

Ex $T: \mathbb{C}^4 \to \mathbb{C}^2$ $\quad$ $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 - 6x_2 + x_3 + 5x_4 \\ 9x_1 + 2x_2 \quad + 6x_4 \end{bmatrix}$

verify (1) & (2)
<u>algebra</u>
(not proofs of closure)

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 \\ 9x_1 \end{bmatrix} + \begin{bmatrix} -6x_2 \\ 2x_2 \end{bmatrix} + \begin{bmatrix} x_3 \\ 0x_3 \end{bmatrix} + \begin{bmatrix} 5x_4 \\ 6x_4 \end{bmatrix}$$

$$= x_1 \begin{bmatrix} 2 \\ 9 \end{bmatrix} + x_2 \begin{bmatrix} -6 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -6 & 1 & 5 \\ 9 & 2 & 0 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

Ex Define $T: \mathbb{C}^2 \to \mathbb{C}^3$ by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 3 & 6 \\ -1 & 4 \\ 9 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Fact $T(\underset{\sim}{x}) = A\underset{\sim}{x}$ is <u>always</u> a linear transformation for any A.

Fact LTTZZ $T: U \to V$ then $T(\underset{\sim}{0}_U) = \underset{\sim}{0}_V$

Proof

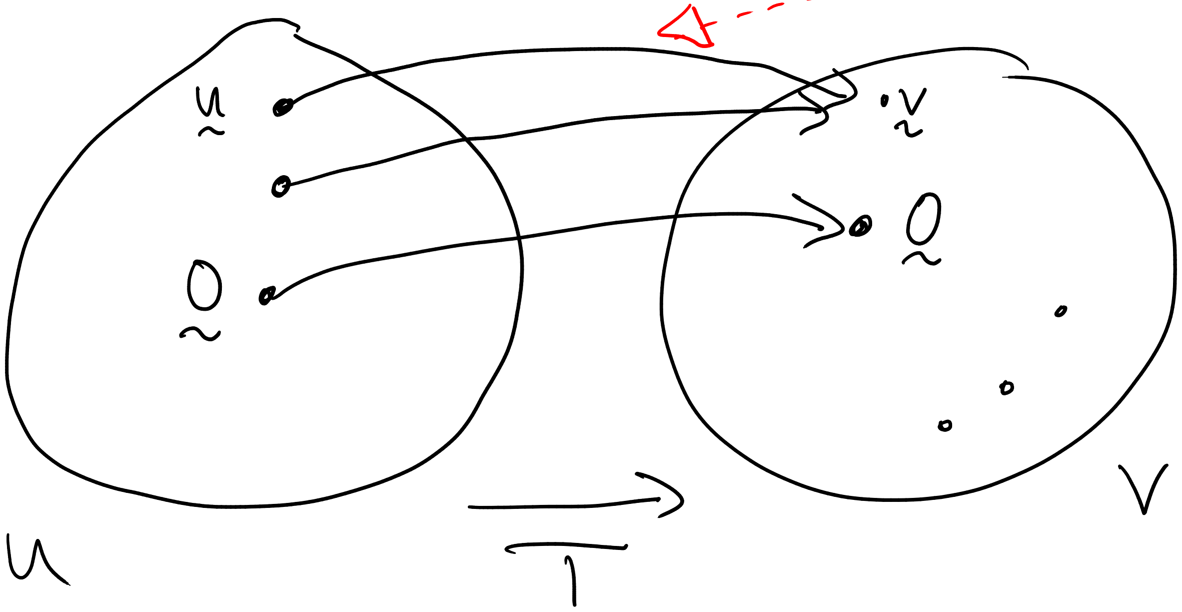
$$\begin{aligned}
\underset{\sim}{0} &= T(\underset{\sim}{0}) - T(\underset{\sim}{0}) \\
&= T(\underset{\sim}{0} + \underset{\sim}{0}) - T(\underset{\sim}{0}) \\
&= T(\underset{\sim}{0}) + T(\underset{\sim}{0}) - T(\underset{\sim}{0}) \\
&= T(\underset{\sim}{0}) + \underset{\sim}{0} \\
&= T(\underset{\sim}{0})
\end{aligned}$$

$$\begin{tikzcd}[column sep=huge, row sep=huge]
\underset{\sim}{u}_1, \underset{\sim}{u}_2 \arrow[r, "T"] \arrow[d, "+_u"'] & T(\underset{\sim}{u}_1), T(\underset{\sim}{u}_2) \arrow[d, "+_v"] \\
\underset{\sim}{u}_1 + \underset{\sim}{u}_2 \arrow[r, "T"'] & \begin{matrix} T(\underset{\sim}{u}_1) + T(\underset{\sim}{u}_2) \\ T(\underset{\sim}{u}_1 + \underset{\sim}{u}_2) \end{matrix}
\end{tikzcd}$$

equal if T is a l.t.

Picture: $T(\alpha \underset{\sim}{u}) = \alpha T(\underset{\sim}{u})$

$T(\underset{\sim}{u}) = \underset{\sim}{v}$



Theorem LTDB

"It is enough to know what a linear transformation does to a basis."