Math 290 B, Friday, April 17 Section ILT

Mon- SLT

Tue- Problem Session

WED- NOTHING

Thu - IVLT

Fri - Problem Session ~~Writing~~

Mon - Writing
- VR

ILT / SLT similar

cut/paste job

<u>Defn</u> $T : U \to V$ then T is <u>injective</u> if whenever $T(\underset{\sim}{u_1}) = T(\underset{\sim}{u_2})$ then $\underset{\sim}{u_1} = \underset{\sim}{u_2}$

Ex $f: \mathbb{R} \to \mathbb{R}$ $f(x) = 5x$ injective

$$f(x_1) = f(x_2)$$
$$5x_1 = 5x_2$$
$$\frac{1}{5}(5x_1) = \frac{1}{5}(5x_2)$$
$$1x_1 = 1x_2$$
$$x_1 = x_2$$

Ex $f(x) = \ln(x)$ injective

$$f(x_1) = f(x_2)$$
$$\ln(x_1) = \ln(x_2)$$
$$e^{\ln(x_1)} = e^{\ln(x_2)}$$
$$x_1 = x_2$$

Ex $g(x) = x^2$

$$g(x_1) = g(x_2)$$
$$x_1^2 = x_2^2$$
$$(-3)^2 = 3^2 \text{ yet } -3 \neq 3$$

not injective

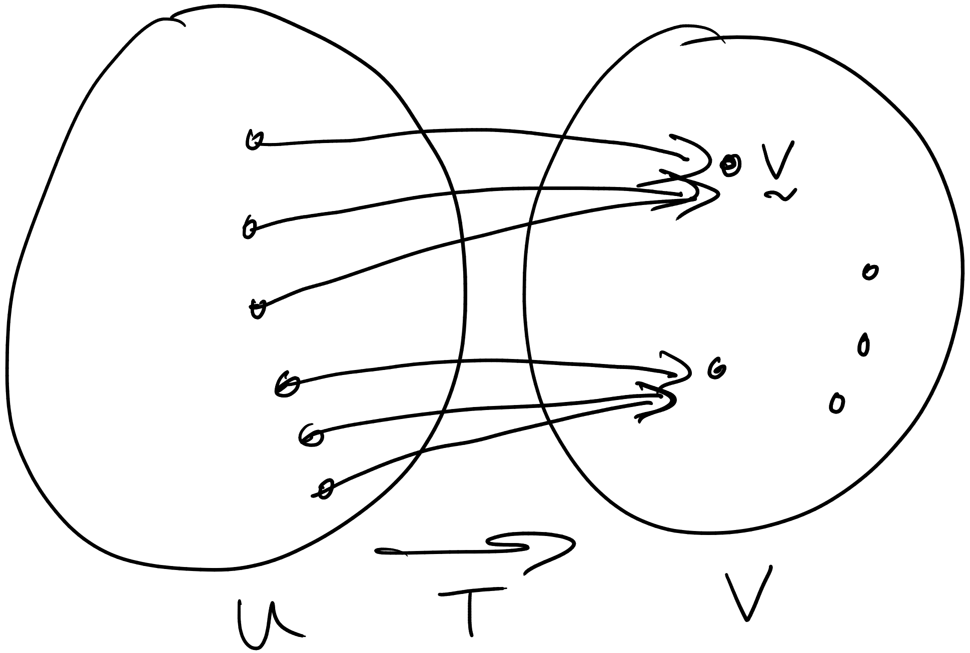
1-1

Ex $T: \mathbb{C}^3 \to \mathbb{C}^4$ $\quad T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} -5x_1 + 4x_2 - 6x_3 \\ -6x_1 + 5x_2 - 7x_3 \\ -x_1 + x_2 - x_3 \\ 3x_1 - 2x_2 + 4x_3 \end{bmatrix}$

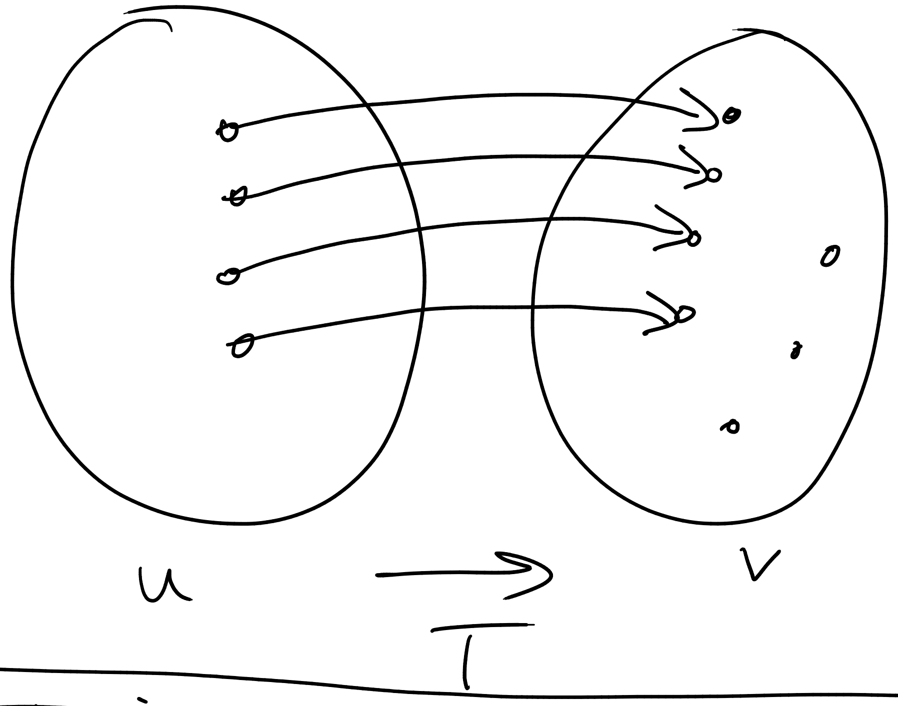
$$\underbrace{T\left(\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}\right)}_{\underset{\sim}{x_1}} = \begin{bmatrix} -11 \\ -13 \\ -2 \\ 7 \end{bmatrix} \qquad \underbrace{T\left(\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right)}_{\underset{\sim}{x_2}} = \begin{bmatrix} -11 \\ -13 \\ -2 \\ 7 \end{bmatrix}$$

$T(\underset{\sim}{x_1}) = T(\underset{\sim}{x_2})$ yet $\underset{\sim}{x_1} \neq \underset{\sim}{x_2}$

not injective



injective



Defn $T: U \to V$, the **kernel** of T is

$$K(T) = \{ \underset{\sim}{u} \in U \mid T(\underset{\sim}{u}) = \underset{\sim}{0} \}$$

Ex $\quad K(T) \quad \uparrow T$

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \underset{\sim}{0}$$

$$\begin{bmatrix} -5x_1 + 4x_2 - 6x_3 \\ -6x_1 + 5x_2 - 7x_3 \\ -x_1 + x_2 - x_3 \\ 3x_1 - 2x_2 + 4x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

homogeneous system
4 equations, 3 variables
coefficient matrix

$$\xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$K(T) = \text{solutions to This system} = N(\quad) = \left\langle \left\{ \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix} \right\} \right\rangle$$

$$T\left(\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right) = T\left(\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}\right) = T\left(\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}\right) + T\left(\begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}\right) = T\left(\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}\right) + \underset{\sim}{0}$$

$$= T\left(\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}\right)$$

Theorem KILT T injective $\iff$ $K(T) = \{\underset{\sim}{0}\}$

Theorem $K(T)$ is a subspace (of U)

Ex $T: P_2 \to M_{22}$ $T(a+bx+cx^2) = \begin{bmatrix} a-2b+c & -a+3b \\ 2b+3c & a-5b-4c \end{bmatrix}$

$K(T) = ?$

$T(a+bx+cx^2) = \underset{\sim}{0}$

$\begin{bmatrix} a-2b+c & -a+3b \\ 2b+3c & a-5b-4c \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

homogeneous system
3 variables, 4 equations,
RREF coefficient matrix

$\xrightarrow{RREF} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

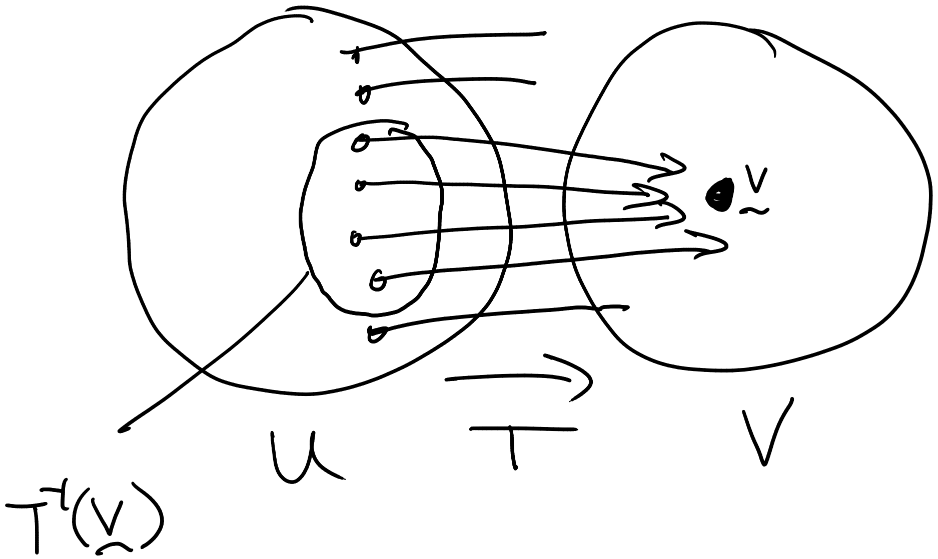
Only solution to system $a=b=c=0$ so $K(T) = \{\underset{\sim}{0}\}$

only vector in kernel $a+bx+cx^2 = 0+0x+0x^2 = \underset{\sim}{0}$

Thus T is injective.

Defn $T: U \to V$, $\underset{\sim}{v} \in V$ $\quad T^{-1}(\underset{\sim}{v}) = \{\underset{\sim}{u} \in U \mid T(\underset{\sim}{u}) = \underset{\sim}{v}\}$

pre-image of $\underset{\sim}{v}$



Theorem KPI Suppose $T(\underset{\sim}{u}) = \underset{\sim}{v}$

then $T^{-1}(\underset{\sim}{v}) = \underset{\sim}{u} + K(T)$

$= \{\underset{\sim}{u} + \underset{\sim}{z} \mid \underset{\sim}{z} \in K(T)\}$