Math 290 B, Monday, April 20 Section SLT

Email: Read-only

Mon: SLT

Tue: Problem Session

Wed: !

Thu: IVLT

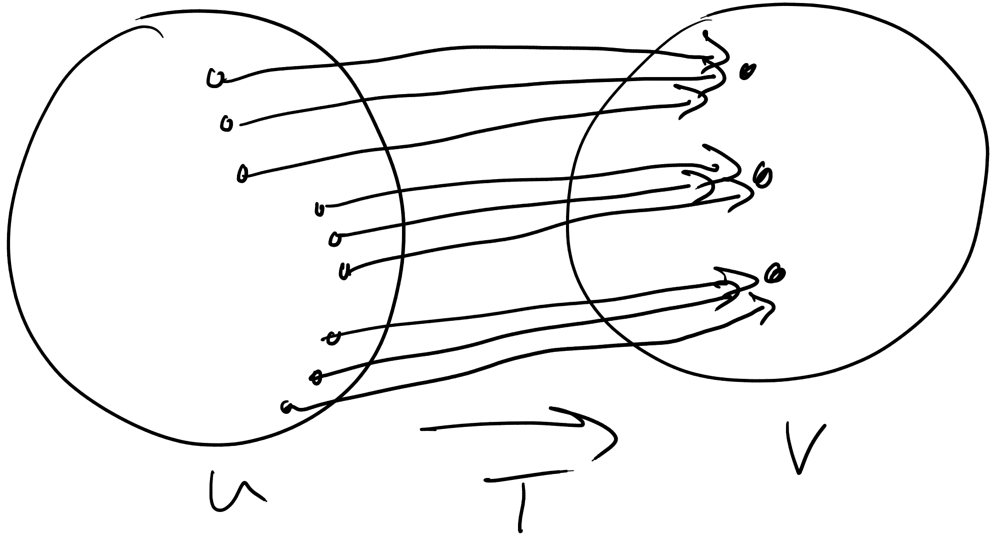
Fri: Problem Session

Mon: VR/RQ (Sage)

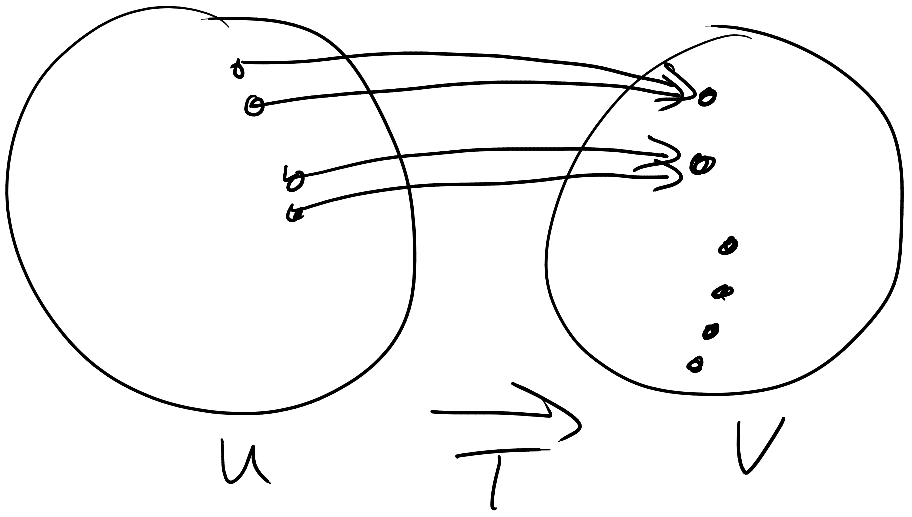
Writing

<u>Defn</u> $T: U \to V$ is <u>surjective</u> if for each $\underset{\sim}{v} \in V$ there is a $\underset{\sim}{u} \in U$ so that $T(\underset{\sim}{u}) = \underset{\sim}{v}$

surjective



non-surjective



Ex $T : P_2 \to P_3$

$T(a+bx+cx^2) = (-5a+4b-6c) + (-6a+5b-7c)x + (-a+b-c)x^2 + (3a-2b+4c)x^3$

Grab $2-x-3x^2+4x^3 \in P_3$, find $a+bx+cx^2 \in P_2$ so that $T(a+bx+cx^2) = 2-x-3x^2+4x^3$?

$(-5a+4b-6c)+(-6a+5b-7c)x + (-a+b-c)x^2 + (3a-2b+4c)x^3 = 2-x-3x^2+4x^3$.

Equate coefficient, 4 equations, 3 variables, non-homogenous system

$$\left[\begin{array}{ccc|c} -5 & 4 & -6 & 2 \\ -6 & 5 & -7 & -1 \\ -1 & 1 & -1 & -3 \\ 3 & -2 & 4 & 4 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

RCLS $\Rightarrow$ no solution so there is no $a+bx+cx^2$.

$\Rightarrow$ T not surjective.

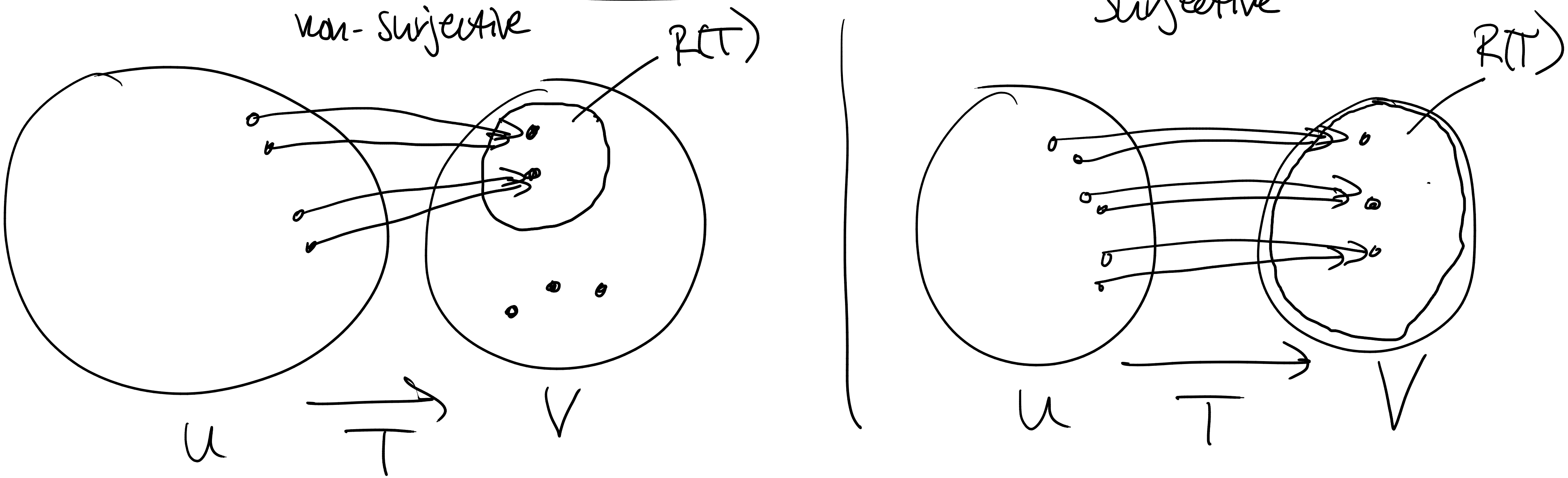
$T^{-1}(2-x-3x^2+4x^3) = \emptyset$

Defn The range of $T: U \to V$ is

$$R(T) = \{ T(\underset{\sim}{u}) \mid \underset{\sim}{u} \in U \} \subseteq V$$

Fact $R(T)$ is a subspace of V.

Theorem RSLT $\quad T$ surjective $\iff R(T) = V$



<u>Theorem SSRLT</u> Suppose $T: U \to V$ & $B = \{\underset{\sim}{u_1}, \underset{\sim}{u_2}, \ldots, \underset{\sim}{u_p}\}$ ~~is~~ (basis?) a spanning set for U. Let $C = \{T(\underset{\sim}{u_1}), T(\underset{\sim}{u_2}), \ldots, T(\underset{\sim}{u_p})\}$ then C is a spanning set for $R(T)$.

<u>Ex</u> $T: M_{22} \to P_2$ $\quad T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a+b+c+4d) + (-a+2c-5d)x + (2a+3b+6c+6d)x^2$

Spanning set of M_{22} (basis!) $= \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\} = B$

$C = \left\{ T\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right), T\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\right), T\left(\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\right), T\left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right) \right\} = \{1 - x + 2x^2, 1 + 3x^2, 1 + 2x + 6x^2, 4 - 5x + 6x^2\}$

So $R(T) = \langle C \rangle$ $\quad$ C is linearly dependent, 4 vectors from P_2, w/ $\dim(P_2) = 3$ by Goldilocks Theorem.

$\alpha_1(1 - x + 2x^2) + \alpha_2(1 + 3x^2) + \alpha_3(1 + 2x + 6x^2) + \alpha_4(4 - 5x + 6x^2) = \underset{\sim}{0} = 0 + 0x + 0x^2$

homogeneous system. 4 variables, 3 equations (HMVEI) $\quad$ coefficient matrix $\xrightarrow{RREF}$ $\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{bmatrix}$ $\quad$ non-trivial solution: $\alpha_4 = 1$

$\alpha_1 = -3, \quad \alpha_2 = -2, \quad \alpha_3 = 1$

$$1(4-5x+6x^2) = 3(1-x+2x^2) + 2(1+3x^2) + (-1)(1+2x+6x^2)$$

$$R(T) = \langle C \rangle = \langle \underbrace{\{1-x+2x^2,\ 1+3x^2,\ 1+2x+6x^2\}}_{\text{basis}} \rangle$$

$$\left.\begin{aligned} &\dim(R(T)) = 3 \\ &\dim(P_2) = 3 \\ &R(T) \subseteq P_2 \end{aligned}\right| \implies R(T) = P_2$$

$$\implies T \text{ surjective.}$$