Math 290B, Tuesday, April 21 Problem Session

Thu - IVLT

Fri - Problem Session

Mon - VR (RQ)
Writing

Tue - MR

Wed - Exam LT

LT.M10

$T: \mathbb{C}^4 \to \mathbb{C}^3$ $\quad S: \mathbb{C}^3 \to \mathbb{C}^2$ $\quad$ MLTCV

$$A = \begin{bmatrix} -1 & 3 & 1 & 9 \\ 2 & 0 & 1 & 7 \\ 4 & 2 & 1 & 2 \end{bmatrix} \qquad T(\underset{\sim}{x}) = A\underset{\sim}{x}$$

$$B = \begin{bmatrix} 1 & -2 & 3 \\ 5 & 4 & 2 \end{bmatrix} \qquad S(\underset{\sim}{x}) = B\underset{\sim}{x}$$

$$S\circ T : \mathbb{C}^4 \to \mathbb{C}^2 \qquad (S\circ T)(\underset{\sim}{x}) = S(T(\underset{\sim}{x}))$$

$$(S\circ T)\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = S\left(T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right)\right) = S\left(\begin{bmatrix} -x_1 + 3x_2 + x_3 + 9x_4 \\ 2x_1 \quad + x_3 + 7x_4 \\ 4x_1 + 2x_2 + x_3 + 2x_4 \end{bmatrix}\right)$$

$$= \begin{bmatrix} (-x_1 + 3x_2 + x_3 + 9x_4) - 2(2x_1 + x_3 + 7x_4) + 3(4x_1 + 2x_2 + x_3 + 2x_4) \\ 5(-x_1 + 3x_2 + x_3 + 9x_4) + 4(2x_1 + x_3 + 7x_4) + 2(4x_1 + 2x_2 + x_3 + 2x_4) \end{bmatrix}$$

$$= \begin{bmatrix} 7x_1 + 9x_2 + 2x_3 + x_4 \\ 11x_1 + 19x_2 + 11x_3 + 77x_4 \end{bmatrix} \qquad C = \begin{bmatrix} 7 & 9 & 2 & 1 \\ 11 & 19 & 11 & 77 \end{bmatrix} \qquad (S\circ T)(\underset{\sim}{x}) = C\underset{\sim}{x}$$

$$C\underset{\sim}{x} = (S\circ T)(\underset{\sim}{x}) = S(T(\underset{\sim}{x})) = S(A\underset{\sim}{x}) = B(A\underset{\sim}{x}) = (BA)\underset{\sim}{x} \Rightarrow C = BA$$

for all $\underset{\sim}{x}$

<u>Check:</u>

$$BA = \underset{2\times 3}{\begin{bmatrix} 1 & -2 & 3 \\ 5 & 4 & 2 \end{bmatrix}} \underset{3\times 4}{\begin{bmatrix} -1 & 3 & 1 & 9 \\ 2 & 0 & 1 & 7 \\ 4 & 2 & 1 & 2 \end{bmatrix}} \underset{\text{EMP}}{=} \begin{bmatrix} 7 & 9 & 2 & 1 \\ 11 & 19 & 11 & 77 \end{bmatrix}$$

LT. T20 Theorem LTLC

$$T(\alpha_1 \underset{\sim}{u_1} + \alpha_2 \underset{\sim}{u_2} + \cdots + \alpha_p \underset{\sim}{u_p}) = \alpha_1 T(\underset{\sim}{u_1}) + \alpha_2 T(\underset{\sim}{u_2}) + \cdots + \alpha_p T(\underset{\sim}{u_p})$$

Definition LT $\Rightarrow$ $\Longleftrightarrow$

1) $T(\underset{\sim}{u} + \underset{\sim}{v}) = T(\underset{\sim}{u}) + T(\underset{\sim}{v})$

2) $T(\alpha \underset{\sim}{u}) = \alpha T(\underset{\sim}{u})$

Make Theorem LTLC a <u>definition</u> of a L.T. $\Longrightarrow$ Definition LT

Prove $\Longrightarrow$ (since $\Longleftarrow$ is Theorem LTLC)

$(\Longrightarrow)$

$$T(\underset{\sim}{u} + \underset{\sim}{v}) = T(1\underset{\sim}{u} + 1\underset{\sim}{v}) = 1\,T(\underset{\sim}{u}) + 1\,T(\underset{\sim}{v}) = T(\underset{\sim}{u}) + T(\underset{\sim}{v})$$

$p=2,\ \alpha_1 = \alpha_2 = 1$

$$T(\alpha \underset{\sim}{u}) = \alpha T(\underset{\sim}{u})$$

$p=1,\ \alpha_1 = \alpha$