Math 290 B, Thursday, April 23 Section IVLT

Fri - Problem Session

Mon - VR (RQ)
WRITING

Tue - MR

Wed - Exam LT

**Defn** $T: U \to V$ **invertible** if there exists $S: V \to U$ so that (1) $(S \circ T)(\underset{\sim}{u}) = I_U(\underset{\sim}{u}) = \underset{\sim}{u}$ (2) $(T \circ S)(\underset{\sim}{v}) = I_V(\underset{\sim}{v}) = \underset{\sim}{v}$

Call $S = T^{-1}$ $\qquad S \circ T = I_U \qquad T \circ S = I_V$

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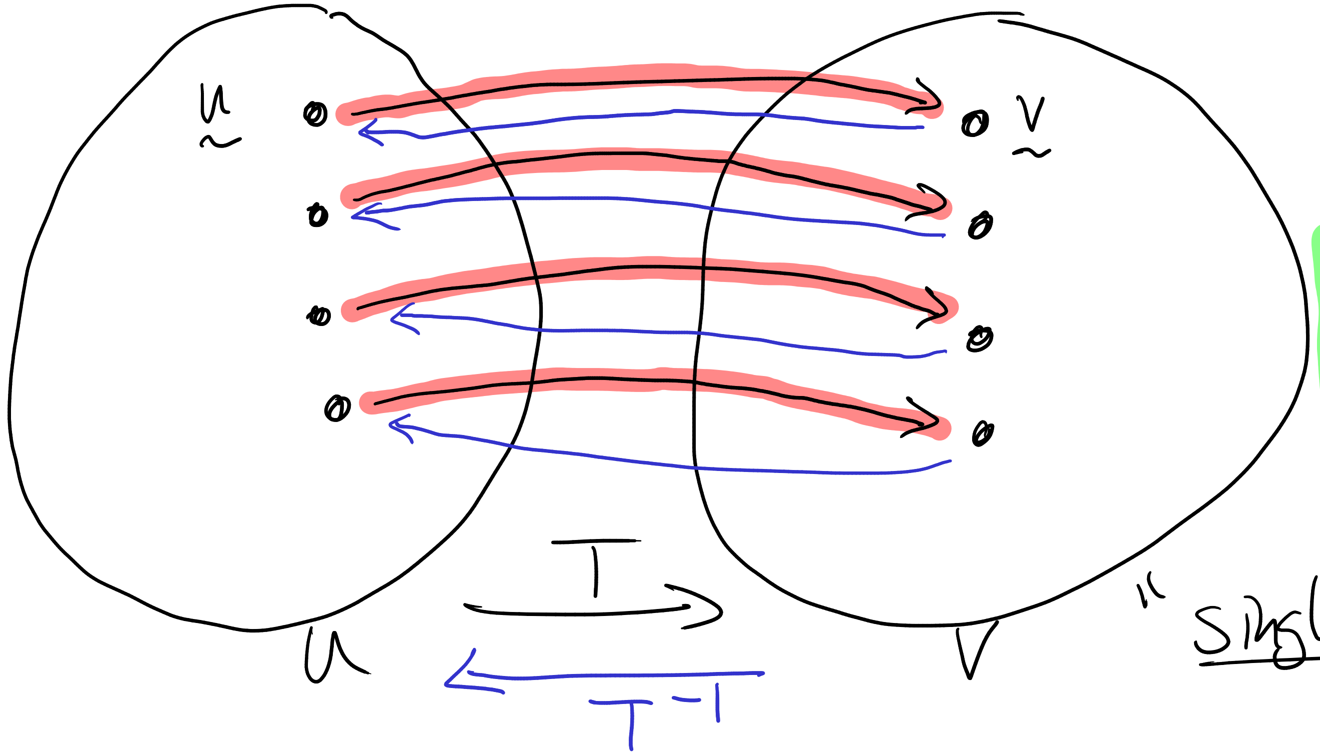
**Theorem ILTIS** $\quad T$ invertible $\iff$ $T$ injective & $T$ surjective

**Proof** ($\Leftarrow$)

Assume $T$ injective & surjective

Blue arrows are $T^{-1}: V \to U$




$T(\underset{\sim}{u}) = \underset{\sim}{v}$

$\boxed{T^{-1}(\underset{\sim}{v})} = \{\boxed{\underset{\sim}{u}}\}$

$\boxed{T^{-1}(\underset{\sim}{v})} = \boxed{\underset{\sim}{u}}$

"singleton"

ex $T : P_3 \to M_{22}$  $T(a+bx+cx^2+dx^3) = \begin{bmatrix} a+b+c+d & 2a+3b+4c-d \\ -a-b+d & 2a+3b+5c+2d \end{bmatrix}$

$K(T) = \{0+0x+0x^2+0x^3\} \Rightarrow n(T) = 0 \Rightarrow T$ injective

$R(T) = M_{22} \Rightarrow r(T) = 4 = \dim(M_{22}) \Rightarrow T$ surjective

$T^{-1} = ???$

$T^{-1} : M_{22} \to P_3$

Grab a basis of $M_{22}$, $B = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$,

Preimages of $B$:

$T^{-1}\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right)$: $T(a+bx+cx^2+dx^3) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} a+b+c+d & 2a+3b+4c-d \\ -a-b+d & 2a+3b+5c+2d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

augmented matrix RREF $\to$ $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 10 \\ 0 & 1 & 0 & 0 & -11 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right]$

$T^{-1}\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\right)$: $\longrightarrow = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \longrightarrow = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \longrightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 & -6 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right]$

$T^{-1}\left(\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\right)$: $\longrightarrow \left[\begin{array}{c} 7 \\ -9 \\ 3 \\ -1 \end{array}\right]$

$T^{-1}\left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right)$: $\longrightarrow \left[\begin{array}{c} -6 \\ 7 \\ -2 \\ 1 \end{array}\right]$

"Looks like" CNM

$T^{-1}\left(\begin{bmatrix}1&0\\0&0\end{bmatrix}\right)=\{10-11x+3x^2-x^3\}$ $\quad T^{-1}\left(\begin{bmatrix}0&1\\0&0\end{bmatrix}\right)=\{5-6x+2x^2-x^3\}$ all singletons

$T^{-1}\left(\begin{bmatrix}0&0\\1&0\end{bmatrix}\right)=\{7-9x+3x^2-x^3\}$ $\quad T^{-1}\left(\begin{bmatrix}0&0\\0&1\end{bmatrix}\right)=\{-6+7x-2x^2+x^3\}$ (exactly one element)

$$T^{-1}\left(\begin{bmatrix}a&b\\c&d\end{bmatrix}\right)=T^{-1}\left(a\begin{bmatrix}1&0\\0&0\end{bmatrix}+b\begin{bmatrix}0&1\\0&0\end{bmatrix}+c\begin{bmatrix}0&0\\1&0\end{bmatrix}+d\begin{bmatrix}0&0\\0&1\end{bmatrix}\right)$$ Theorem VRRB

$$=a\,T^{-1}\left(\begin{bmatrix}1&0\\0&0\end{bmatrix}\right)+b\,T^{-1}\left(\begin{bmatrix}0&1\\0&0\end{bmatrix}\right)+c\,T^{-1}\left(\begin{bmatrix}0&0\\1&0\end{bmatrix}\right)+d\,T^{-1}\left(\begin{bmatrix}0&0\\0&1\end{bmatrix}\right)$$ Theorem LTLC

$$=a(10-11x+3x^2-x^3)+b(5-6x+2x^2-x^3)+c(7-9x+3x^2-x^3)+d(-6+7x-2x^2+x^3)$$

$$=(10a+5b+7c-6d)+(-11a-6b-9c+7d)x+(3a+2b+3c-2d)x^2+(-a-b-c+d)x^3$$

Check: $(T\circ T^{-1})\left(\begin{bmatrix}a&b\\c&d\end{bmatrix}\right)=\begin{bmatrix}a&b\\c&d\end{bmatrix}$

$(T^{-1}\circ T)(a+bx+cx^2+dx^3)=a+bx+cx^2+dx^3$