Math 290B, Monday, April 27 Section VR

Mon - VR
writing

Tue - MR

Wed - Exam LT

**Defn** $V$ - vector space, $\underset{\sim}{v} \in V$

$B = \{\underset{\sim}{u}_1, \underset{\sim}{u}_2, \dots, \underset{\sim}{u}_n\}$. Then

$\underset{\sim}{v} = a_1 \underset{\sim}{u}_1 + a_2 \underset{\sim}{u}_2 + \dots + a_n \underset{\sim}{u}_n$ (Theorem VRRB)

Then $\rho_B(\underset{\sim}{v}) = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$

**Fact** $\rho_B : V \to \mathbb{C}^n$ is a bijective linear transformation

"bijection" = injective + surjective

$\underline{Ex}$ $\mathbb{C}^3$ $\quad \underset{\sim}{v} = \begin{bmatrix} -12 \\ 9 \\ 16 \end{bmatrix} \quad B = \left\{ \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ 7 \end{bmatrix} \right\}$

$$\rho_B(\underset{\sim}{v}) = \rho_B\left(\begin{bmatrix} -12 \\ 9 \\ 16 \end{bmatrix}\right) = \rho_B\left(2\begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix} + (-1)\begin{bmatrix} 4 \\ 3 \\ -2 \end{bmatrix} + 2\begin{bmatrix} -2 \\ 3 \\ 7 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$$

$$C = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$$\rho_C(\underset{\sim}{v}) = \rho_C\left(\begin{bmatrix} -12 \\ 9 \\ 16 \end{bmatrix}\right) = \rho_C\left(-12\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 9\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + 16\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -12 \\ 9 \\ 16 \end{bmatrix}$$

$\underline{Ex}$ $M_{22}$, $\quad \underset{\sim}{w} = \begin{bmatrix} 8 & 7 \\ 23 & 21 \end{bmatrix}, \quad B = \left\{ \begin{bmatrix} -2 & 4 \\ 3 & 3 \end{bmatrix}, \begin{bmatrix} 0 & -2 \\ 3 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 5 \\ 3 & 5 \end{bmatrix}, \begin{bmatrix} 3 & 1 \\ 5 & 5 \end{bmatrix} \right\}$

$$\rho_B\left(\begin{bmatrix} 8 & 7 \\ 23 & 21 \end{bmatrix}\right) = \rho_B\left(2\begin{bmatrix} -2 & 4 \\ 3 & 3 \end{bmatrix} + 0\begin{bmatrix} 0 & -2 \\ 3 & -1 \end{bmatrix} + (-1)\begin{bmatrix} 0 & 5 \\ 3 & 5 \end{bmatrix} + 4\begin{bmatrix} 3 & 1 \\ 5 & 5 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 0 \\ -1 \\ 4 \end{bmatrix}$$

$$D = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

$$\rho_D(\underset{\sim}{w}) = \rho_D\left( \begin{bmatrix} 8 & 7 \\ 23 & 21 \end{bmatrix} \right) = \begin{bmatrix} 8 \\ 7 \\ 23 \\ 21 \end{bmatrix}$$

<u>Ex</u> $\rho_B : M_{22} \to \mathbb{C}^4 \qquad \rho_B^{-1} : \mathbb{C}^4 \to M_{22}$

$$\rho_B^{-1}\left( \begin{bmatrix} 1 \\ 0 \\ 2 \\ -1 \end{bmatrix} \right) = 1 \begin{bmatrix} -2 & 4 \\ 3 & 3 \end{bmatrix} + 0 \begin{bmatrix} 0 & -2 \\ 3 & -1 \end{bmatrix} + 2 \begin{bmatrix} 0 & 5 \\ 3 & 5 \end{bmatrix} + (-1) \begin{bmatrix} 3 & 1 \\ 5 & 5 \end{bmatrix} = \begin{bmatrix} -5 & 13 \\ 4 & 8 \end{bmatrix}$$

<u>Theorem</u> If $\dim(U) = \dim(V)$ then $U$ and $V$ are isomorphic.

<u>Ex</u> $\dim(P_{22}) = 23$ & $\dim(\mathbb{C}^{23}) = 23$ so $P_{22} \cong \mathbb{C}^{23}$

Coordinatization Principle

(Section MR)

**Defn** $T: U \to V$, $B = \{\underset{\sim}{u}_1, \underset{\sim}{u}_2, \dots, \underset{\sim}{u}_n\}$ basis of $U$, $C$ basis of $V$

$$M^T_{B,C} = \left[\rho_C(T(\underset{\sim}{u}_1)) \,\middle|\, \rho_C(T(\underset{\sim}{u}_2)) \,\middle|\, \cdots \,\middle|\, \rho_C(T(\underset{\sim}{u}_n))\right] \qquad A\underset{\sim}{x} = \lambda \underset{\sim}{x}$$

**Ex** $T: P_2 \to P_2$ $\quad B = C = \{1-2x+x^2,\ 1-3x+x^2,\ x-x^2\}$

$$\rho_C(T(\underset{\sim}{u}_1)) = \rho_C(T(1-2x+x^2)) = \rho_C(3-6x+3x^2) = \rho_C(3(1-2x+x^2)+0(1-3x+x^2)+0(x-x^2)) = \begin{bmatrix}3\\0\\0\end{bmatrix}$$

$$\rho_C(T(\underset{\sim}{u}_2)) = \rho_C(1-3x+x^2) = \rho_C(0(1-2x+x^2)+1(1-3x+x^2)+0(x-x^2)) = \begin{bmatrix}0\\1\\0\end{bmatrix}$$

$$\rho_C(T(\underset{\sim}{u}_3)) = \rho_C(-x+x^2) = \rho_C(0(1-2x+x^2)+0(1-3x+x^2)+(-1)(x-x^2)) = \begin{bmatrix}0\\0\\-1\end{bmatrix}$$

$$\boxed{T(a+bx+cx^2) = (5a+2b+2c)+(-10a-3b-2c)x+(6a+2b+c)x^2}$$

$$M^T_{B,C} = \begin{bmatrix}3 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -1\end{bmatrix}$$