Math 290B Friday, May 1 Problem Session

Mon - CB (RQ)
- Sage

Tue - Problem Session
Housekeeping
Writing 11 AM

Wed - Exam R

Final Exam

Tuesday, May 12

9 AM - two hour design/three hour limit

(8 AM by appointment)

MR.T40

All $T: U \to V$ set $\mathcal{LT}(U,V)$ | $\mathcal{LT}(P_7, M_{2,3})$

$$\rho \in \mathcal{LT}(P_7, M_{2,3}) \qquad \rho(a_0 + a_1 x + \cdots + a_7 x^7) = \begin{bmatrix} 0 & a_3 & 0 \\ a_1 + a_6 & 0 & -a_4 \end{bmatrix}$$

$\dim U = n \qquad \dim V = m$

$$\Rightarrow \quad \mathcal{LT}(U,V) \cong M_{mn} \qquad \Big| \qquad \mathcal{LT}(\underset{8}{P_7}, \underset{6}{M_{23}}) \cong M_{6,8}$$

linear transformation

Proof Need an isomorphism, ϕ

$$\phi : \mathcal{LT}(U,V) \to M_{mn} \qquad \phi(T) = M^T_{B,C}$$

Choose B basis of U $\qquad T: U \to V$
C basis of V

Check Need ϕ injective, surjective, operation-preserving (2 properties of a LT)

$S: U \to V$

$$\phi(T+S) = M^{T+S}_{B,C} = M^T_{B,C} + M^S_{B,C} = \phi(T) + \phi(S)$$

MRSLT

$(T+S)(u) = T(u) + S(u)$

Defn LTA

$\ker \phi$?

$m \times n$ zero matrix

$\ker \phi = \{ T \mid \phi(T) = \underset{\sim}{O} \}$

So $T: U \to V$

$T(\underset{\sim}{u}) = \underset{\sim}{0}$ (Exercises Z)

T (aka Z) is the zero vector of $\mathcal{L}(U,V)$.

$T \in \ker \phi$

$T(\underset{\sim}{u}) = \rho_C^{-1}\left(M^T_{B,C}\, \rho_B(\underset{\sim}{u})\right)$ FTMR

$= \rho_C^{-1}\left(O\, \rho_B(\underset{\sim}{u})\right) = \rho_C^{-1}\left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}\right)$

$= 0(\ ) + 0(\ ) + \cdots + 0(\ ) = \underset{\sim}{0}_V$

So $\ker \phi = \{Z\}$ trivial

$\underset{\text{KILT}}{\Longrightarrow}$ ϕ injective