Math 491 Friday March 13

Eigenvalues & Eigenvectors A square

Section SD / FCLA

$A\underset{\sim}{x} = \lambda \underset{\sim}{x}$

MVP

A & B similar $\iff$ there is S, $S^{-1}AS = B$

Similarity is an equivalence relation

Example DAB

Example NDMS4

Theorem DC $\quad A_{n\times n}$ diagonalizable $\iff$ there is a linearly independent set of n eigenvectors for A, B

(*) forms a basis of $\mathbb{C}^n$ so that a matrix representation of $T: \mathbb{C}^n \to \mathbb{C}^n$, $T(\underset{\sim}{x}) = A\underset{\sim}{x}$ is a diagonal matrix

$M^T_{B,B} \leftarrow$ diagonal

Proof $(\Longleftarrow)$ A $\quad \{x_1, x_2, \ldots, x_n\}$ n lin. ind. eigenvectors of A for $\lambda_1, \lambda_2, \ldots, \lambda_n$

$$S = [x_1 \mid x_2 \mid \cdots \mid x_n].$$

$$\cancel{(*)} \quad AS = A[x_1 \mid \cdots \mid x_n] = [Ax_1 \mid Ax_2 \mid \cdots \mid Ax_n]$$

$$= [\lambda_1 x_1 \mid \lambda_2 x_2 \mid \cdots \mid \lambda_n x_n]$$

$$= [\lambda_1 Se_1 \mid \lambda_2 Se_2 \mid \cdots \mid \lambda_n Se_n]$$

$$= [S(\lambda_1 e_1) \mid S(\lambda_2 e_2) \mid \cdots \mid S(\lambda_n e_n)]$$

$$= S[\lambda_1 e_1 \mid \lambda_2 e_2 \mid \cdots \mid \lambda_n e_n]$$

$$= S \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \lambda_3 & \\ & & & \ddots & \\ & & & & \lambda_n \end{bmatrix} = SD$$

$$[e_1 \mid e_2 \mid \cdots \mid e_n] = I_n \qquad e_k = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \leftarrow \text{slot } k$$

$$Se_2 = [x_1 \mid x_2 \mid \cdots \mid x_n] \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$= 0x_1 + 1x_2 + 0x_3 + \cdots + 0x_n$$

$$= x_2$$

$$\Longrightarrow AS = SD$$

$$S^{-1}AS = D$$

SCLA (1.4)

Defn $T: V \to V$ lin. transf. W subspace of V.

W is an <u>invariant</u> (T-invariant) subspace of V for T

if $T(\underset{\sim}{w}) \in W$ for all $\underset{\sim}{w} \in W$.

<u>Example</u> Every eigenspace is an invariant subspace.
(Theorem 1.4.5)

A $n \times n$ matrix, λ eigenvalue $\underset{\sim}{x} \neq \underset{\sim}{0}$ $A\underset{\sim}{x} = \lambda \underset{\sim}{x} \Rightarrow (A - \lambda I)\underset{\sim}{x} = \underset{\sim}{0} \Rightarrow$

$\underset{\sim}{x} \in N(A - \lambda I) = \mathcal{E}_A(\lambda)$

$T: \mathbb{C}^n \to \mathbb{C}^n$, $T(\underset{\sim}{x}) = A\underset{\sim}{x}$

$\underset{\sim}{u} \in \mathcal{E}_A(\lambda)$ $T(\underset{\sim}{u}) = \lambda \underset{\sim}{u} \in \mathcal{E}_A(\lambda)$

$\uparrow$ scalar closure

<u>Next Time</u> Example 1.4.2

$$\left[\begin{array}{c|c} 3 & 0 \\ \hline 0 & 3 \end{array}\right]$$