Math 491 Thursday, March 26 Chapter 20 Problem Session

$\mathbb{C}^n$ $\mathbb{R}^n$ F^n

9) $W = \{ f : [0,1] \to \underline{\mathbb{R}} \mid f(0) = 0 \}$ subspace $C[0,1]$ $Ax = \lambda x$

1) $W \neq \emptyset$ $\quad f(x) = 0$ for all x $\quad f \in W$ $\quad f = \underset{\sim}{0}$ $(f = z)$

2) $f, g \in W$ know $f(0) = 0$ & $g(0) = 0$. Check $f+g$.

$(f+g)(0) = f(0) + g(0) = 0 + 0 = 0$. So $f+g \in W$

3) $f \in W$, $\alpha \in \underline{\mathbb{R}}$ Know $f(0) = 0$. Check αf

$(\alpha f)(0) = \alpha f(0) = \alpha \cdot 0 = 0$ so $\alpha f \in W$

18/ V, W v.sp. over F $\quad \mathrm{Hom}(V,W) = \{ T \mid T: V \to W \}$

$(S+T): V \to W \qquad (S+T)(\underset{\sim}{v}) = S(\underset{\sim}{v}) +_W T(\underset{\sim}{v})$

$(\alpha S): V \to W \qquad (\alpha S)(\underset{\sim}{v}) = \alpha S(\underset{\sim}{v})$

$\underset{\sim}{0}$? $\qquad Z: V \to W \qquad Z(\underset{\sim}{v}) = \underset{\sim}{0}_W$

$(S+Z)(\underset{\sim}{v}) = S(\underset{\sim}{v}) + Z(\underset{\sim}{v}) = S(\underset{\sim}{v}) + \underset{\sim}{0} = S(\underset{\sim}{v})$ for <u>all</u> $\underset{\sim}{v}$

So $S+Z = S$ $\qquad f(x) = x^2 \quad g(x) = x \cdot x \quad f = g$

<u>+ commutative?</u>

$(S+T)(\underset{\sim}{v}) = S(\underset{\sim}{v}) + T(\underset{\sim}{v}) = T(\underset{\sim}{v}) + S(\underset{\sim}{v}) = (T+S)(\underset{\sim}{v})$ for <u>all</u> $\underset{\sim}{v}$

So $S+T = T+S$

18 (Cont) (b) V vector space over F, $V^* = \mathrm{Hom}(V, F)$ F'

Ex $V = \mathbb{R}^3$ $T: \mathbb{R}^3 \to \mathbb{R}$ $T\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = \underline{a+b+c}$ $(= [a+b+c])$

— $\phi_2\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = \phi_2(a\underset{\sim}{e_1} + b\underset{\sim}{e_2} + c\underset{\sim}{e_3})$
$= b$

$B = \{\underset{\sim}{v_1}, \dots, \underset{\sim}{v_n}\}$ basis of V

$\phi_i : V \to F$, $\phi_i(\underset{\sim}{v}) = \phi_i(\alpha_1 \underset{\sim}{v_1} + \alpha_2 \underset{\sim}{v_2} + \dots + \alpha_n \underset{\sim}{v_n}) = \alpha_i$

Claim: $\phi_1, \phi_2, \dots, \phi_n$ basis of V^*.

L.I. $\rho_1 \phi_1 + \rho_2 \phi_2 + \dots + \rho_n \phi_n = \underset{\sim}{0} = Z$

So $(\rho_1 \phi_1 + \dots + \rho_n \phi_n)(\underset{\sim}{v_k}) = Z(\underset{\sim}{v_k}) = \underset{\sim}{0} = 0$

$\rho_1 \phi_1(\underset{\sim}{v_k}) + \rho_2 \phi_2(\underset{\sim}{v_k}) + \dots + \rho_n \phi_n(\underset{\sim}{v_k}) = \underset{\sim}{0}$ | for any k.

$\rho_1 0 + \rho_2 0 + \dots + \rho_k 1 + \dots + \rho_n 0 = \underset{\sim}{0}$

$\rho_k = 0$