Math 491, Monday, April 27 Chapter 23

Tue - Sage

Thu - 23/ Quintic

Fri - Problem Session
Sage 23

Mon - Exam 4 22/23

Tue - Early Start (<u>8 AM</u>)
3x Presentations
20 min + 5 min questions

Final Tuesday 10AM Pacific

Defn Fixed field F, $G =$ some group of automorphisms of F (not <u>all</u>)

$$F_G = \{ f \in F \mid \sigma(f) = f \text{ for all } \sigma \in G \}$$

Fact F_G subfield of F (F extension of F_G)

Theorem E is the splitting field of F for a seperable polynomial. Then $E_{G(E/F)} = F$.

inverse process

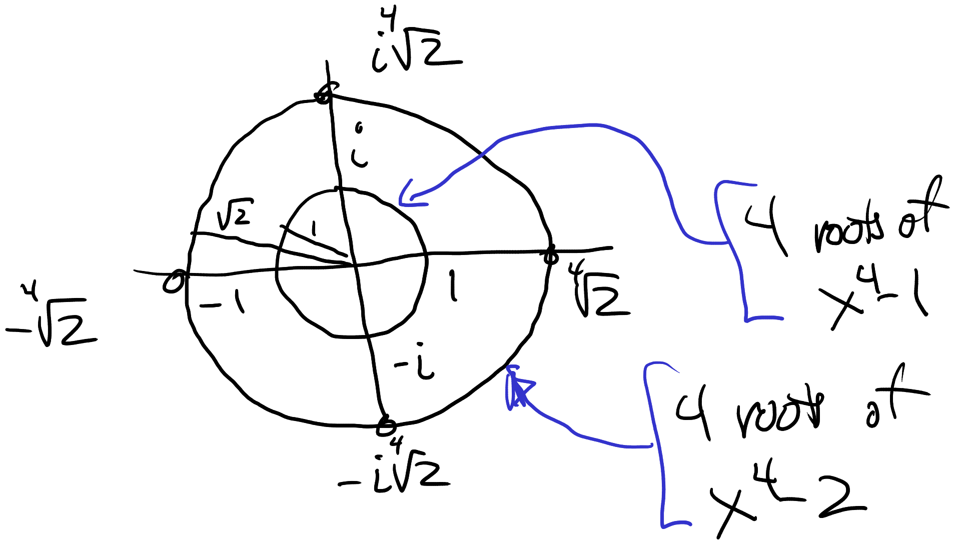
Theorem $F = E_G$ for some group G of automorphisms of E

$$\Rightarrow [E:F] \leq |G|$$

Defn An extension E/F is <u>normal</u> if whenever an irreducible polynomial over F has <u>a</u> root in E, then <u>all</u> of its roots are in E.

Ex Extension $\mathbb{Q}(2^{1/4})/\mathbb{Q}$.

Irreducible polynomial x^4-2 roots



$\mathbb{Q}(2^{1/4})$ has two roots of x^4-2, $(\pm\sqrt[4]{2})$, but does not contain two other roots $(\pm i\sqrt[4]{2})$

So $\mathbb{Q}(2^{1/4})/\mathbb{Q}$ is not a normal extension

The splitting field of x^4-2 is $\mathbb{Q}(2^{1/4}, i2^{1/4})$

$\mathbb{Q}(2^{1/4}, i2^{1/4})$

4 [2 | $\mathbb{Q}(2^{1/4})$

2 | $\mathbb{Q}$

Galois group

$$\sigma: \begin{aligned} i2^{1/4} &\to i2^{1/4} \\ -i2^{1/4} &\to -i2^{1/4} \\ 2^{1/4} &\to -2^{1/4} \\ -2^{1/4} &\to 2^{1/4} \end{aligned}$$

σ has order 2

Full Galois group $\cong D_4$

$\langle\sigma\rangle$ not normal in D_4

<u>Theorem</u> The following are equivalent for extension E/F.

1) E finite normal seperable extension.

2) E splitting field over F of a seperable polynomial

3) $F = E_G$ for some group G of automorphisms of E.

Fundamental Theorem of Galois Theory

F finite field or has characteristic zero. E finite normal extension

trivial

$$\begin{tikzcd}
E \arrow[r] \arrow[d, dash] & G(E/E) \arrow[d, dash] \\
K \arrow[r] \arrow[d, dash] & G(E/K) \arrow[d, dash] \\
F \arrow[r] & G(E/F)
\end{tikzcd}$$

subfields

subgroups

normal?

normal

$[K:F] = [G(E/F):G(E/K)]$

INFINITE

big

FINITE

inclusion-reversing

$\Rightarrow$ bijection from fields (subfields) to subgroups

K normal extension of F

$\Longleftrightarrow$ $G(E/K)$ is a normal subgroup of $G(E/F)$

If so,

$$G(K/F) \cong \frac{G(E/F)}{G(E/K)}$$