

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points. Partial credit is proportional to the quality of your explanation. You may use Sage to row-reduce matrices, except in the question that asks you to row-reduce without Sage. No other use of Sage may be used as justification for your answers. When you use Sage be sure to explain your input and show any relevant output (rather than just describing salient features).

1. Solve the following two systems of linear equations and express the solutions for each as a set of column vectors. (30 points)

(a)

$$\begin{aligned} -2x_1 + 2x_3 + 6x_4 - 2x_5 &= 7 \\ 4x_1 - 3x_2 + 3x_3 + 3x_4 + 5x_5 &= 10 \\ -5x_1 + 4x_2 - 3x_3 - x_4 - 5x_5 &= -5 \\ -3x_1 + x_2 + x_3 + 5x_4 - 3x_5 &= 18 \end{aligned}$$

(b)

$$\begin{aligned} -2x_1 + 5x_2 - 8x_3 &= 5 \\ x_1 - 3x_2 + 5x_3 &= -3 \\ x_1 - x_2 + 2x_3 &= 0 \\ -x_1 + 4x_2 - 3x_3 &= 8 \end{aligned}$$

2. Without using Sage, find a matrix  $B$  in reduced row-echelon form which is row-equivalent to  $A$ . It is especially important to show all of your work, so it is clear you have not used Sage. (15 points)

$$A = \begin{bmatrix} 1 & -1 & 3 & 3 \\ 2 & -1 & 5 & 6 \\ 2 & 0 & 4 & 6 \end{bmatrix}$$



3. Is the matrix  $B$  singular or not? Provide a justification for your answer. (15 points)

$$B = \begin{bmatrix} -5 & 2 & -3 & 7 & 2 \\ -1 & 2 & 2 & 7 & 2 \\ -2 & 1 & -1 & 3 & 1 \\ 1 & 1 & 3 & 5 & 1 \\ -3 & 3 & -2 & -1 & 3 \end{bmatrix}$$

4. Compute the null space of the matrix  $C$ ,  $\mathcal{N}(C)$ . (15 points)

$$C = \begin{bmatrix} 1 & 2 & 8 & -8 & -2 \\ 0 & 1 & 2 & -2 & -3 \\ 0 & -2 & -4 & 4 & 7 \end{bmatrix}$$



5. Each statement below is true or false. State which and give a **justification**. Answers without justifications will receive no credit. A good justification for “True” might be a theorem, and a good justification for “False” might be a simple counterexample (an example which demonstrates that the statement is not always true). (25 points)

(a) A homogeneous system is consistent.

(b) A system with a nonsingular coefficient matrix has the zero vector as a solution.

(c) A consistent system with 12 variables and 8 equations has infinitely many solutions.

(d) A system with a singular coefficient matrix has no solutions.

(e) A system with 3 variables and 2 equations has infinitely many solutions.

