1. Is the matrix $B$ unitary? Why or why not? (15 points)

$$B = \begin{bmatrix}
2 & 1 & -26 \\
3 & 2 & 17 \\
1 & -8 & 1
\end{bmatrix}$$

2. Find the solution set of the linear system $\mathcal{LS}(A, b)$ using the inverse of the coefficient matrix. No credit will be given for solutions obtained by other methods. (15 points)

$$A = \begin{bmatrix}
-1 & 0 & 1 & 0 \\
-3 & 1 & 5 & -8 \\
2 & 0 & -3 & 4 \\
-2 & 1 & 3 & -3
\end{bmatrix} \quad b = \begin{bmatrix}
5 \\
6 \\
-5 \\
8
\end{bmatrix}$$
3. Consider the matrix $A$. (40 points)

$$\begin{bmatrix}
3 & 14 & -16 & 41 & 60 & 23 \\
1 & 5 & -6 & 15 & 22 & 9 \\
-1 & -7 & 11 & -25 & -36 & -20 \\
-5 & -22 & 28 & -71 & -100 & -45 \\
1 & 6 & -9 & 21 & 30 & 16
\end{bmatrix}$$

(a) Find a linearly independent set $S$, whose span is the column space of $A$, $\langle S \rangle = C(A)$, and whose elements are each a column of $A$.

(b) Find a linearly independent set $T$, whose span is the column space of $A$, $\langle T \rangle = C(A)$, by using the matrix $L$ from the extended echelon form of $A$.

(c) Find a linearly independent set $R$, whose span is the column space of $A$, $\langle R \rangle = C(A)$, by using theorems about the row space of a matrix.

(d) Find a linearly independent set $U$, whose span is the row space of $A$, $\langle U \rangle = R(A)$.

(e) Construct a nonzero vector $b$ from one of the sets $S$, $T$, $R$, $U$ (your choice, but say which you are using) and explain how you know that $LS(A, b)$ has a solution (without simply solving the system).
4. Suppose that $A$ is an $m \times n$ matrix, and $O_{n \times p}$ and $O_{m \times p}$ are zero matrices of the indicated sizes. Give a careful proof that $A O_{n \times p} = O_{m \times p}$. (15 points)

5. Suppose that $A$ is a nonsingular matrix. Prove that $LS(A, b)$ has a unique solution by first assuming there are two solutions (Proof Technique U), and also using a representation of the system with a matrix-vector product (Theorem SLEMM). Full-credit requires following these suggestions, so in particular, do not simply quote existing theorems to provide a simple one-line proof. (15 points)