

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points. Partial credit is proportional to the quality of your explanation. You may use Sage to row-reduce matrices. No other use of Sage may be used as justification for your answers. When you use Sage be sure to explain your input and show any relevant output (rather than just describing salient features).

1. Is the matrix  $B$  unitary? Why or why not? (15 points)

$$B = \begin{bmatrix} 2 & 1 & -26 \\ 3 & 2 & 17 \\ 1 & -8 & 1 \end{bmatrix}$$

2. Find the solution set of the linear system  $\mathcal{LS}(A, \mathbf{b})$  using the inverse of the coefficient matrix. No credit will be given for solutions obtained by other methods. (15 points)

$$A = \begin{bmatrix} -1 & 0 & 1 & 0 \\ -3 & 1 & 5 & -8 \\ 2 & 0 & -3 & 4 \\ -2 & 1 & 3 & -3 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 5 \\ 6 \\ -5 \\ 8 \end{bmatrix}$$



3. Consider the matrix  $A$ . (40 points)

$$A = \begin{bmatrix} 3 & 14 & -16 & 41 & 60 & 23 \\ 1 & 5 & -6 & 15 & 22 & 9 \\ -1 & -7 & 11 & -25 & -36 & -20 \\ -5 & -22 & 28 & -71 & -100 & -45 \\ 1 & 6 & -9 & 21 & 30 & 16 \end{bmatrix}$$

(a) Find a linearly independent set  $S$ , whose span is the column space of  $A$ ,  $\langle S \rangle = \mathcal{C}(A)$ , and whose elements are each a column of  $A$ .

(b) Find a linearly independent set  $T$ , whose span is the column space of  $A$ ,  $\langle T \rangle = \mathcal{C}(A)$ , by using the matrix  $L$  from the extended echelon form of  $A$ .

(c) Find a linearly independent set  $R$ , whose span is the column space of  $A$ ,  $\langle R \rangle = \mathcal{C}(A)$ , by using theorems about the row space of a matrix.

(d) Find a linearly independent set  $U$ , whose span is the row space of  $A$ ,  $\langle U \rangle = \mathcal{R}(A)$ .

(e) Construct a nonzero vector  $\mathbf{b}$  from **one** of the sets  $S, T, R, U$  (your choice, but say which you are using) and explain how you know that  $\mathcal{LS}(A, \mathbf{b})$  has a solution (without simply solving the system).



4. Suppose that  $A$  is an  $m \times n$  matrix, and  $\mathcal{O}_{n \times p}$  and  $\mathcal{O}_{m \times p}$  are zero matrices of the indicated sizes. Give a careful proof that  $A\mathcal{O}_{n \times p} = \mathcal{O}_{m \times p}$ . (15 points)

5. Suppose that  $A$  is a nonsingular matrix. Prove that  $\mathcal{LS}(A, \mathbf{b})$  has a unique solution by first assuming there are two solutions (Proof Technique U), and also using a representation of the system with a matrix-vector product (Theorem SLEMM). Full-credit requires following these suggestions, so in particular, do not simply quote existing theorems to provide a simple one-line proof. (15 points)

