Math 290 Name: Dr. Beezer
Exam 4 Spring 2021
Chapter VS

Show *all* of your work and *explain* your answers fully. There is a total of 100 possible points.
Partial credit is proportional to the quality of your explanation. You may use Sage to row-reduce matrices. No other use of Sage may be used as justification for your answers. When you use Sage be sure to explain your input and show any relevant output (rather than just describing salient features).

1. Determine if the following subset of M_{22}, the vector space of 2×2 matrices, is linearly independent. (15 points)

$$C = \left\{ \begin{bmatrix} 1 & 0 \\ 3 & -4 \end{bmatrix}, \begin{bmatrix} -4 & 1 \\ -7 & 8 \end{bmatrix}, \begin{bmatrix} 3 & -1 \\ 5 & -2 \end{bmatrix} \right\}$$

2. Row-reduce the following matrix with Sage. Based on this computation, and with no further computations at all (by hand, nor by Sage), specify the rank and nullity of the matrix. Then determine, with explanation, the nicest possible basis for the column space of A, $\mathcal{N}(A)$. (15 points)

$$A = \begin{bmatrix} 1 & -1 & -7 & -2 & -2 \\ 0 & 1 & 2 & -1 & -4 \\ 0 & 2 & 4 & -1 & -6 \end{bmatrix}$$

3. Illustrate the use of Theorem TSS to determine that W is a subspace of the vector space $\mathbb{C}^2$. (20 points)

$W = \left\{ \begin{bmatrix} a \\ b \end{bmatrix} \middle| \, a + 2b = 0 \right\}$

4. Find a basis for the following subspace of P_2, the vector space of polynomials with degree at most 2. (20 points)

$X = \left\{ a + bx + cx^2 \middle| \, a - b + 5c = 0, \; 2a + b + c = 0 \right\}$

5. Suppose that V is a vector space, and $\mathbf{v} \in V$. Prove that $0\mathbf{v} = \mathbf{0}$. (15 points)

6. Suppose that $D = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \ldots, \mathbf{v}_k\} \subseteq \mathbb{C}^m$ is a linearly independent set, and A is a nonsingular $m \times m$ matrix. Prove that $E = \{A\mathbf{v}_1, A\mathbf{v}_2, A\mathbf{v}_3, \ldots, A\mathbf{v}_k\} \subseteq \mathbb{C}^m$ is a linearly independent set. (15 points)