Class—FCLA OD, Cholesky Decomposition

Advanced Linear Algebra

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A random 6×6 matrix.

```
A = random_matrix(ZZ, 6, 6, x=-9, y=9) + I*
    random_matrix(ZZ, 6, 6, x=-9, y=9)
A = A.change_ring(CDF)
A
```

Check the base ring, avoid the `Symbolic Ring`.

```
A.base_ring()
```

Multiply A by its adjoint to form B. Hermitian (self-adjoint). Positive definite, since the eigenvalues are all positive. So a Cholesky decomposition is possible.

```
B = A.conjugate_transpose()*A
B.round(2)
```

```
B.eigenvalues()
```

```
U = B.cholesky()
U.round(2)
```

And the check.

```
U*U.conjugate_transpose()
```

Do it again, but exactly this time, over the rationals.

```
C = random_matrix(ZZ, 6, 6, x=-9, y=9).change_ring(QQ)
C
```

Multiply C by its adjoint to form D. Hermitian (self-adjoint). Positive definite, since the eigenvalues are all positive. So a Cholesky decomposition is possible.

```
D = C.conjugate_transpose()*C
D
```

```
D.eigenvalues()
```

```
Q = D.cholesky()
Q
```

And the check.

```
Q*Q.conjugate_transpose()
```

Sage's `.indefinite_factorization()` will use *only* field operations, giving exact results by avoiding square roots. The cost is an intermediate diagonal matrix D, so the factorization is LDL^*.

```
L, DD = D.cholesky()
DD
```

```
L
```