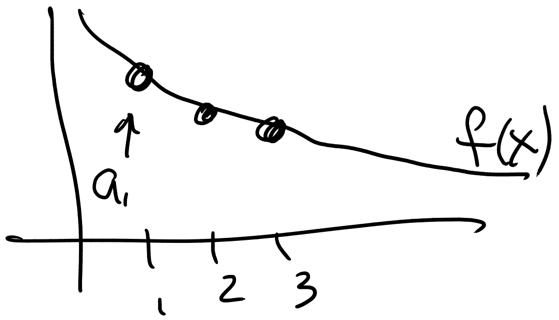
Math 181 Monday, March 22 Section 10.4

Tue
Thu
Fri
] 10.5 / 10.6

BYOB Music Performers

Integral test requires $a_n$ be decreasing.

$\underline{a_n > a_{n+1}}$ $f(n) = a_n$




$a_n$ decreasing $\iff f'(x) < 0$

$16 - 7x^3$

Alternating Series Tests

- Integral test, comparison test assume $\sum_{n=1}^{\infty} a_n$ $\underline{\underline{\text{with}}}$ $a_n \geq 0$.
- Now handle / analyze $\sum_{n=1}^{\infty} (-1)^n b_n$ with $b_n \geq 0$

Ex $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n} = \sum_{n=1}^{\infty} (-1)^n \underbrace{\frac{1}{n}}_{b_n} = -1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} - \cdots$

signs ($\pm$) alternate

Alternating Series Test

$b_n \geq 0$, $b_n$ decreasing, $\displaystyle\lim_{n\to\infty} b_n = 0 \Rightarrow \sum_{n=1}^{\infty} (-1)^n b_n$ converges.

Ex $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges.

① $b_n = \frac{1}{n} \geq 0$

② $n < n+1$

$\frac{1}{n} > \frac{1}{n+1}$

$b_n > b_{n+1}$

$b_n$ decreasing

③ $\displaystyle\lim_{n\to\infty} b_n = 0$

By alternating series test, $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges.

(Note: $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.
p-series, $p=1$
"Harmonic Series")

Ex $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1}$

① $b_n = \frac{1}{2n-1} \geq 0$

② Decreasing

$b_n > b_{n+1}$

$\frac{1}{2n-1} > \frac{1}{2(n+1)-1}$

$2n-1 < 2(n+1)-1$

$2n-1 < 2n+1$

$0 < 2$ ✓

③ $\lim_{n\to\infty} \frac{1}{2n-1} = 0$

Series converges by alternating series test.

Fact: Converges to $\frac{\pi}{4}$

$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} \cdots\cdots = \frac{\pi}{4}$

Ex

$0.\bar{3} = 0.333\ldots\ldots$

$= \sum_{n=1}^{\infty} 3\left(\frac{1}{10}\right)^n = \frac{1}{10} \sum_{k=0}^{\infty} 3\left(\frac{1}{10}\right)^k = \frac{1}{10} \frac{3}{1-\frac{1}{10}} = \frac{1}{10} \frac{3}{\frac{9}{10}} = \frac{3}{9} = \frac{1}{3}$

re-indexing

since $\left|\frac{1}{10}\right| < 1$

Ex $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{3n^2+2n-6}$ Alternating $b_n = \dfrac{n^2}{3n^2+2n-6} \geq 0$

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{n^2}{3n^2+\cancel{2n}-\cancel{6}} = \frac{1}{3} \neq 0$$

So we can't use the alternating series test.

Start over.

$$\lim_{n\to\infty} \frac{(-1)^n n^2}{3n^2+2n-6}$$

bc the sequence oscillates between (roughly) $\frac{1}{3}$ & $\frac{-1}{3}$

Limit does not exist

So this limit $\neq 0$, so by the $n^{th}$-term test for divergence,

the series diverges.

Defn $\sum_{n=1}^{\infty} a_n$ converges & $\sum_{n=1}^{\infty} |a_n|$ converges then $\sum_{n=1}^{\infty} a_n$ is absolutely convergent. Otherwise, conditionally convergent.

Ex $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ converges by alternating series test

$\sum_{n=1}^{\infty} \left|\frac{(-1)^n}{n^2}\right| = \sum_{n=1}^{\infty} \frac{1}{n^2}$ converges as p-series $p=2>1$

absolute convergence

Ex $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges by alternating series test

$\sum_{n=1}^{\infty} \left|\frac{(-1)^n}{n}\right| = \sum_{n=1}^{\infty} \frac{1}{n}$ diverges as p-series $p=1 \not> 1$

conditional convergence