Math 181 Thursday, April 1 Section 10.7

WW 10.6.6

Fri - 10.7/10.8

Mon - 10.8

Tue - 8.1

BYOB Foreign Country

$$\frac{1}{1-4x} = \sum_{n=0}^{\infty} a_n x^n$$

Geometric Series: $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$

$$\frac{1}{1-4x} = \frac{1}{1-(4x)} = \sum_{n=0}^{\infty} 1(4x)^n$$

"r"

$$= \sum_{n=0}^{\infty} 4^n x^n \qquad a_n = 4^n$$

Converges if $|r| < 1 \Rightarrow |4x| < 1 \rightarrow 4|x| < 1 \Rightarrow |x| < \frac{1}{4}$

$-\frac{1}{4} < x < \frac{1}{4}$

10.6.7

$$f(x) = 1 - 5x + 5x^2 + x^3 - 5x^4 + 5x^5 + x^6 + \cdots$$

$$= (1 - 5x + 5x^2) + (x^3 - 5x^4 + 5x^5) + (x^6 - 5x^7 + 5x^8) + \cdots$$

$$= (1 - 5x + 5x^2) + x^3(1 - 5x + 5x^2) + x^6(1 - 5x + 5x^2) + \cdots$$

$$= (1 - 5x + 5x^2)\left[1 + x^3 + x^6 + x^9 + x^{12} + \cdots\right]$$

$$= (1 - 5x + 5x^2)\left[1 + x^3 + (x^3)^2 + (x^3)^3 + (x^3)^4 + \cdots\right]$$

Geometric w/ $r = x^3$

$$= (1 - 5x + 5x^2) \quad \frac{1}{1 - x^3}$$

Ex

$$\frac{1}{1+x^2} = \frac{1}{1-(-x^2)} = 1 + (-x^2) + (-x^2)^2 + (-x^2)^3 + (-x^2)^4 + \cdots$$

Geometric w/ $r = -x^2$

$$= 1 - x^2 + x^4 - x^6 + x^8 + \cdots$$

Converges for $|r| = |-x^2| < 1 \rightarrow |x|^2 < 1 \rightarrow -1 < x < 1$

Anti differentiate both sides

$$\int_0^1 \frac{dx}{1+x^2} = \int_0^1 1 - x^2 + x^4 - x^6 + \cdots \, dx = \int_0^1 \sum_{n=0}^{\infty} (-1)^n x^{2n} \, dx$$

FTC

$$\arctan(x)\Big|_0^1 = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \Big|_0^1 = \sum_{n=0}^{\infty} \int_0^1 (-1)^n x^{2n} \, dx$$

$$\arctan(1) - \arctan(0) = \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\right) - 0$$

$$\frac{\pi}{4} - 0 = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

$$\pi = 4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\right)$$

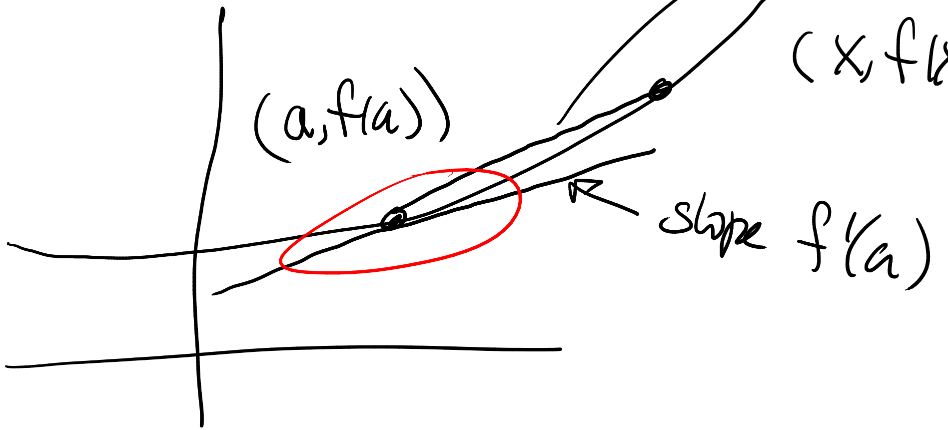
Converges (alternating series test) SLOWLY

10.7 Taylor Polynomials

(finite polynomials, not infinite series)

Approximate $f(x)$ by a polynomial

Tangent line



$$\text{slope} = \frac{f(x)-f(a)}{x-a} \cong f'(a)$$

$$\frac{f(x)-f(a)}{x-a} \cong f'(a) \rightarrow f(x)-f(a) \cong f'(a)(x-a)$$

$$f(x) \cong \underbrace{f(a)+f'(a)\underbrace{(x-a)}_{\Delta x}}_{\text{tangent line}}$$

Good linear approximation

Good quadratic approximation???

$f(x)$, approximate w/ $h(x) = ex^2 + fx + g$ $\quad e, f, g$?

Match derivatives at 0.

$$f(0) = h(0) = e\,0^2 + f \cdot 0 + g = g \Rightarrow g = f(0)$$

$$f'(0) = h'(0) = (2ex + f)\big|_{x=0} = 2e \cdot 0 + f = f \Rightarrow f = f'(0)$$

$$f''(0) = h''(0) = 2e\big|_{x=0} = 2e \Rightarrow 2e = f''(0)$$

$$e = \tfrac{1}{2} f''(0)$$

$$h(x) = \underbrace{\tfrac{1}{2} f''(0)\,x^2}_{\text{new}} + \underbrace{f'(0)\,x + f(0)}_{\text{tangent line at } a=0}$$

Ex $f(x) = e^x$, quadratic approximation at $x=0$

$f'(x) = e^x$, $f''(x) = e^x$

$$h(x) = \frac{1}{2} f''(0) x^2 + f'(0) x + f(0)$$

$$= \frac{1}{2} e^0 x^2 + e^0 x + e^0$$

$$= \frac{1}{2} \cdot 1 \; x^2 + 1 \cdot x + 1 = \frac{1}{2} x^2 + x + 1$$