Math 181 Friday, April 9 Section 8.2

Mon - Review

Tue - Exam 2
Chapter 10

Thu - 9.1

Fri - 9.1/9.2

WW 8.1.1

$$p(x) = C\, x(4-x) \quad 0 \le x \le 3$$

$$1 = \int_0^3 C x(4-x)\,dx = C\int_0^3 x(4-x)\,dx$$

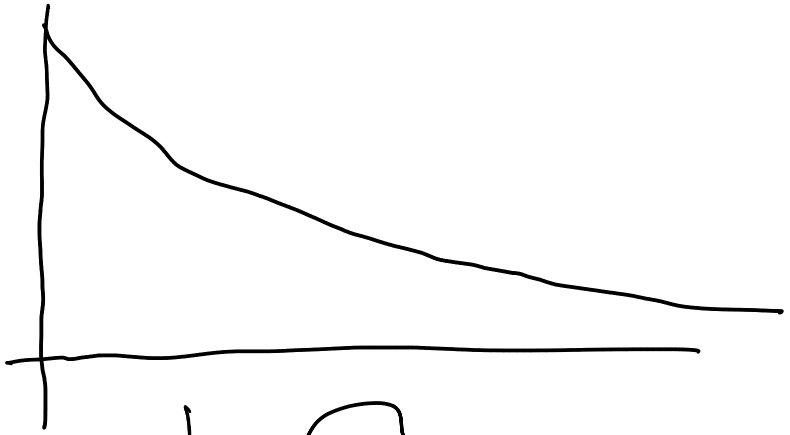
WW 8.1.4

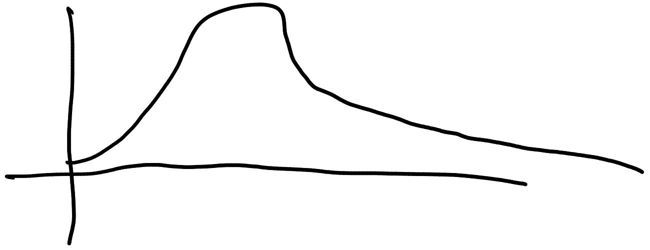
$$f(x) = \frac{1}{\alpha} e^{-x/\alpha} \quad 0 \le x \le \infty$$

$$P(X \geq 60) = 0.50$$

$$\int_{60}^{\infty} \frac{1}{\alpha} e^{-x/\alpha}\,dx = 0.50$$

$$\Rightarrow \alpha = 60/\ln 2$$

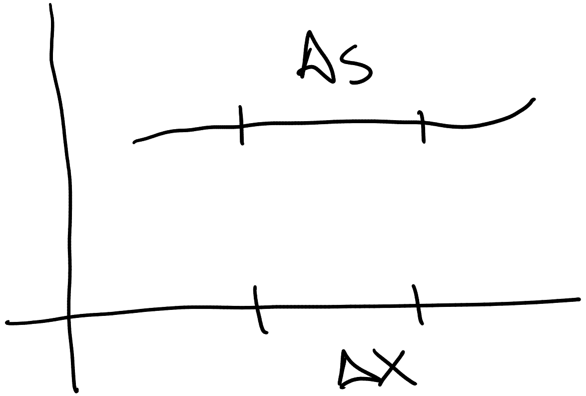




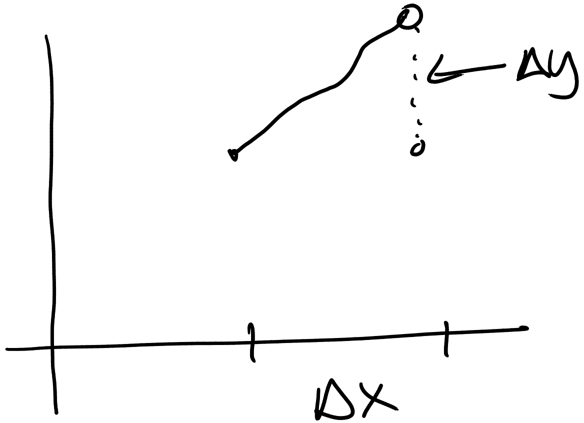
$$ds/dx = \sqrt{1+(f'(x))^2}$$

$$ds = \sqrt{1+(f'(x))^2}\ dx$$

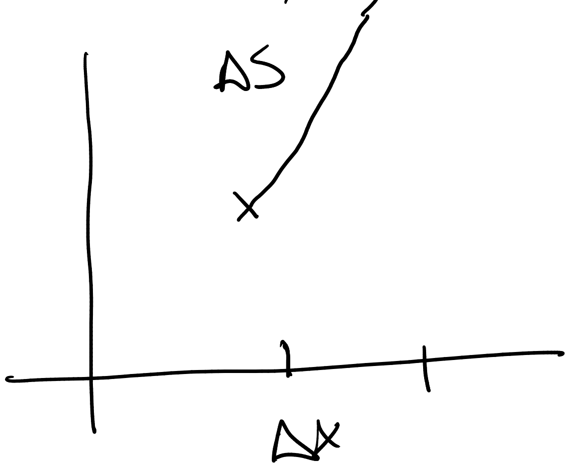
$$\frac{dy}{dx} = \frac{dy}{\cancel{du}}\cdot\frac{\cancel{du}}{dx}$$



$f' = 0 \qquad \sqrt{1+0^2} = 1$



$f' = 1 \qquad \sqrt{1+1^2} = \sqrt{2}$



$f' =$ huge

$\sqrt{1+(f')^2} \simeq \sqrt{f'^2} = f'$

Ex Length of $y = x^{3/2}$ on $3 \le x \le 4$

$dy/dx = \frac{3}{2} x^{1/2}$ $\qquad ds = \sqrt{1 + \left(\frac{3}{2}x^{1/2}\right)^2}\, dx$

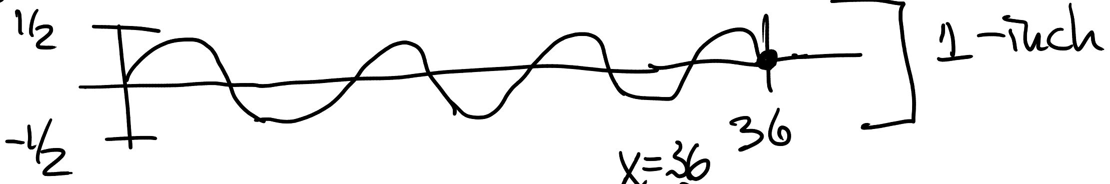
$\qquad = \sqrt{1 + \frac{9}{4}x}\, dx$

$$S = \int ds = \int_{x=3}^{x=4} \sqrt{1 + \frac{9}{4}x}\, dx$$ u-substitution $u = 1 + \frac{9}{4}x$

$$= \frac{80\sqrt{10}}{27} - \frac{31\sqrt{31}}{27} \cong 2.977$$

Ex Corrugated tin sheets, cross-section $y = \frac{1}{2}\sin(\pi x)$, $0 \le x \le 36$ inches

side view



How long is the original flat piece of tin?

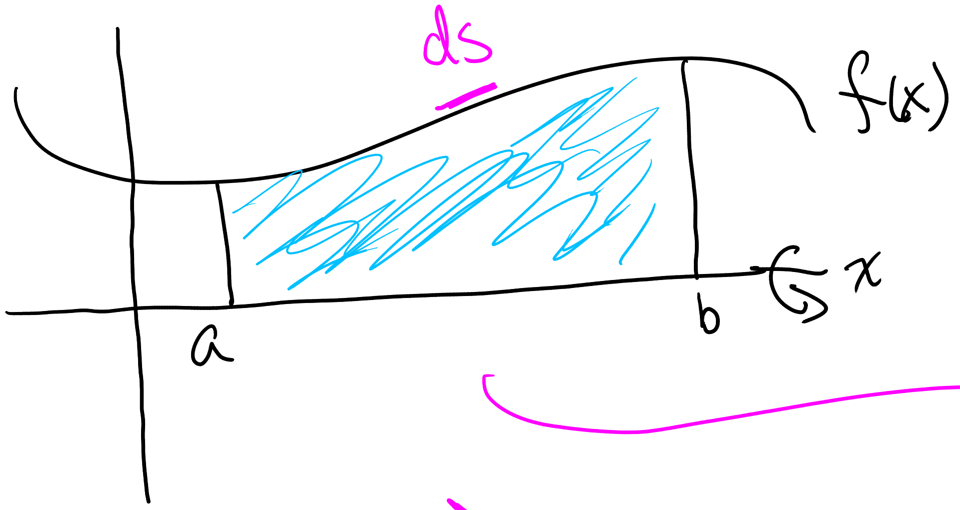
$\frac{dy}{dx} = \frac{\pi}{2}\cos(\pi x)$ $\qquad S = \int_{x=0}^{x=36} \sqrt{1 + \left(\frac{\pi}{2}\cos(\pi x)\right)^2}\, dx = 52.690$ inches

not easy to antidifferentiate

numerically via Simpson's Rule

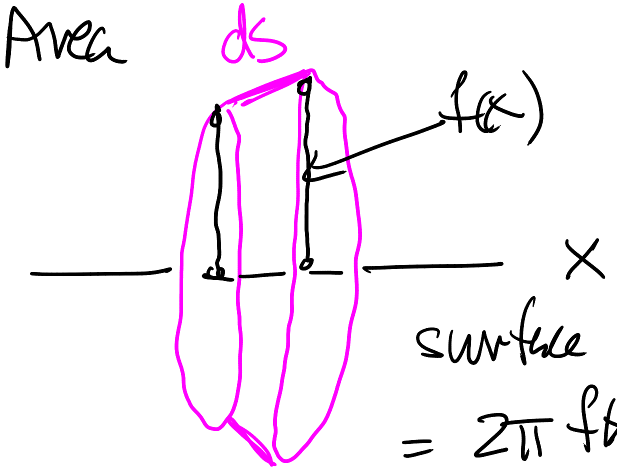
Surface Area

Solid of revolution, rotate $y=f(x)$ around x-axis, $a \le x \le b$



~~Volume?~~

Surface Area

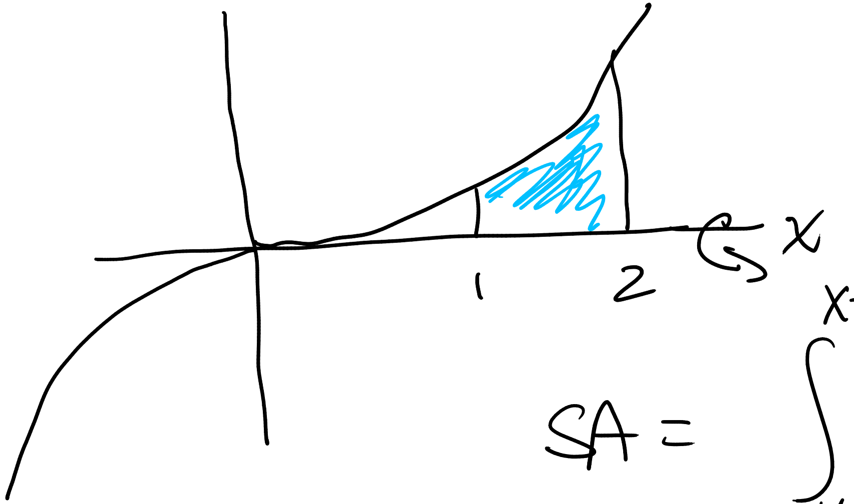


surface area $= \underbrace{2\pi f(x)}_{\text{circumference}} \underbrace{ds}_{\text{depth}}$

$$SA = \int 2\pi f(x)\, ds$$

$$= \int 2\pi f(x) \sqrt{1+(f'(x))^2}\, dx$$

Ex Rotate $y = x^3$ around x axis for $1 \le x \le 2$. Surface area?



$$dy/dx = 3x^2$$

$$ds = \sqrt{1 + (3x^2)^2}\, dx = \sqrt{1 + 9x^4}\, dx$$

$$SA = \int_{x=1}^{x=2} 2\pi x^3 \sqrt{1 + 9x^4}\, dx \simeq 199.48$$

$u = 1 + 9x^4$

u-substitution

Ex $y = \frac{1}{6}x^3 + \frac{1}{2x}$, ds?

$$= \frac{1}{6}x^3 + \frac{1}{2}x^{-1}$$

$$\frac{dy}{dx} = \frac{1}{2}x^2 - \frac{1}{2}x^{-2}$$

$$ds = \sqrt{1 + \left(\tfrac{1}{2}x^2 - \tfrac{1}{2}x^{-2}\right)^2}\, dx$$

$$= \sqrt{1 + \left(\tfrac{1}{2}x^2\right)^2 \underbrace{- 2\left(\tfrac{1}{2}x^2\right)\left(\tfrac{1}{2}x^{-2}\right)}_{-1/2} + \left(-\tfrac{1}{2}x^{-2}\right)^2}\, dx$$

$$= \sqrt{\left(\tfrac{1}{2}x^2\right)^2 + \tfrac{1}{2} + \left(-\tfrac{1}{2}x^{-2}\right)^2}\, dx$$

$$= \sqrt{\left(\tfrac{1}{2}x^2\right)^2 + 2\left(\tfrac{1}{2}x^2\right)\left(\tfrac{1}{2}x^{-2}\right) + \left(\tfrac{1}{2}x^{-2}\right)^2}\, dx = \sqrt{\left(\tfrac{1}{2}x^2 + \tfrac{1}{2}x^{-2}\right)^2}\, dx$$

$$= \left(\tfrac{1}{2}x^2 + \tfrac{1}{2}x^{-2}\right) dx$$