Math 181 Monday, April 12 Chapter 10 Review

Tue – Exam 3 Chapter 10

Thu – 9.1

Fri – 9.1/9.2

BYOB:

Mon – 9.2

Ex $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+6n}{n^4+3n^2+2}$ $\displaystyle\lim_{n\to\infty} \frac{n^2+6n}{n^4+3n^2+2} = 0$

$\dfrac{n^2+6n}{n^4+3n^2+2}$ like $\dfrac{n^2}{n^4} = \dfrac{1}{n^2}$, $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ p-series, $p=2>1$ $\Rightarrow$ convergence

$$L = \lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{n^2+6n}{n^4+3n^2+2}}{1/n^2} = \lim_{n\to\infty} \frac{n^2(n^2+6n)}{n^4+3n^2+2}$$

$$= \lim_{n\to\infty} \frac{n^4+6n^2}{n^4+3n^2+2} = \lim_{n\to\infty} \frac{(n^4+6n^2)\,1/n^4}{(n^4+3n^2+2)\,1/n^4} = \lim_{n\to\infty} \frac{1+6/n^2}{1+3/n^2+2/n^4} = \frac{1}{1} = 1$$

$L=0$?
$L>0$?
$L=\infty$

$L>0$, $L\neq\infty$, $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^2}$ converges $\Rightarrow \displaystyle\sum_{n=1}^{\infty}\frac{n^2+6n}{n^4+3n^2+2}$ converges also

10.8.7 WW $c = \pi/2$ Taylor Series

$f(x) = \sin(x)$ $f(\pi/2) = 1$

$f'(x) = \cos(x)$ $f'(\pi/2) = 0$

$f''(x) = -\sin(x)$ $f''(\pi/2) = -1$

$f^{(3)}(x) = -\cos(x)$ $f^{(3)}(\pi/2) = 0$

$f^{(4)}(x) = \sin(x)$ $f^{(4)}(\pi/2) = 1$

$f^{(5)}(x) = \cos(x)$ $f^{(5)}(\pi/2) = 0$

$f^{(6)}(x) = -\sin(x)$ $f^{(6)}(\pi/2) = -1$

$f^{(7)}(x) = -\cos(x)$ $f^{(7)}(\pi/2) = 0$

$\frac{1}{2}(x-\pi/2)^2 < 1$

$$1 + \frac{0}{1!}(x-\pi/2)^1 + \frac{-1}{2!}(x-\pi/2)^2 + \frac{0}{3!}(x-\pi/2)^3 + \frac{1}{4!}(x-\pi/2)^4 + \frac{0}{5!}(x-\pi/2)^5 + \frac{-1}{6!}(x-\pi/2)^6 + \frac{0}{7!}(x-\pi/2)^7 + \cdots$$

$$= 1 - \frac{1}{2!}(x-\pi/2)^2 + \frac{1}{4!}(x-\pi/2)^4 - \frac{1}{6!}(x-\pi/2)^6 + \cdots$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!}(x-\pi/2)^{2k}$$

~~radius~~ interval of convergence

$$\rho = \lim_{k\to\infty} \left| \frac{(-1)^{k+1}/2(k+1)!\,(x-\pi/2)^{2(k+1)}}{(-1)^k/(2k)!\,(x-\pi/2)^{2k}} \right|$$

$$= \lim_{k\to\infty} \left| \frac{2k!}{(2k+2)!} \cdot \frac{(x-\pi/2)^{2k+2}}{(x-\pi/2)^{2k}} \right| = \lim_{k\to\infty} \frac{1}{(2k+2)(2k+1)}(x-\pi/2)^2$$

$$= (x-\pi/2)^2 \boxed{\lim_{k\to\infty} \frac{1}{(2k+2)(2k+1)}} = (x-\pi/2)^2 \cdot 0 = 0 < 1$$

so the series converges (absolutely) for all x.