

Math 181

Thursday, April 22

Section 9.3

Numerical ~~Analytical~~ solutions to Diff Eq

Fri - 9.4

BYOB - Free Play

Mon - 11.1

Ex  $\frac{dy}{dx} = \ln(x+y)$  ( $e^y = x+y$ ?)

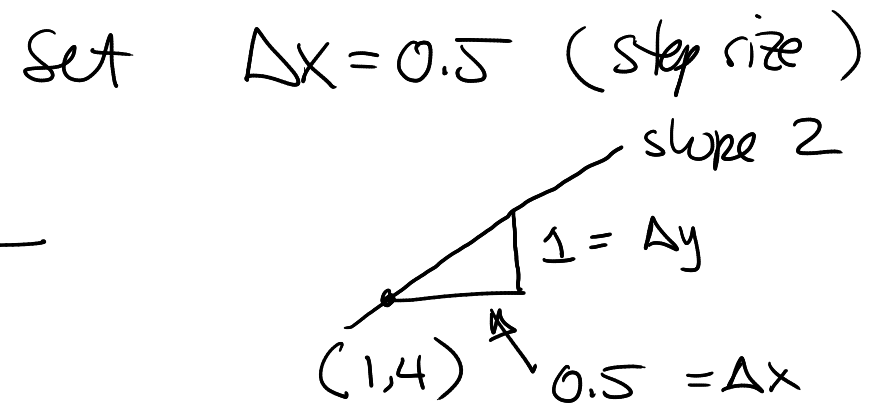
$x+y < 0$  ,  $dy/dx$  not defined

$x+y = 1 \Rightarrow dy/dx = \ln(1) = 0$

Ex Numerical solution to  $\frac{dy}{dx} = 2x$  ,  $y = 4$  when  $x = 1$   
slope information

initial condition

$x$	slope	$\Delta y$	$y$
1			4
1.5	$2(1)=2$	$2 \cdot 0.5=1$	5
2	$2(1.5)=3$	$3(0.5)=1.5$	6.5
2.5	$2(2)=4$	$4(0.5)=2$	8.5
3	$2(2.5)=5$	$5(0.5)=2.5$	11



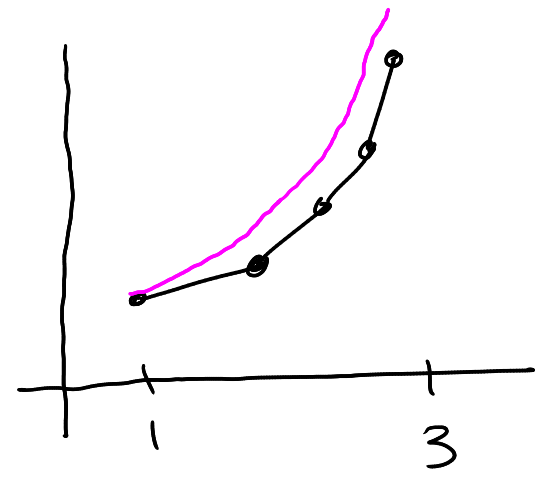
Exact?

$$y' = 2x \Rightarrow y = x^2 + C$$

$$4 = y = 1^2 + C \Rightarrow C = 3$$

$$y = x^2 + 3$$

$$x = 3 \rightarrow y = 3^2 + 9 = 12$$



Given  $y = f(x)$ , we have  $f'(x) \leftarrow$  derivative  
 $f(a)$  known also, initial condition. Find  $f(b)$ .

$$f(b) = f(a) + [f(b) - f(a)]$$

$$= f(a) + \int_a^b f'(x) dx$$

$$\approx f(a) + f'(x_1)\Delta x + f'(x_2)\Delta x_2 + f'(x_3)\Delta x_3 + \dots + f'(x_n)\Delta x$$

height

area of rectangle

width

Ex Numerical solution to  $dy/dx = x+y$ , when  $x=0, y=1$

not separable

Find  $y$  when  $x=3$

Step size  $\Delta x = 1$

$x$	$\frac{dy}{dx}$	$\Delta y$	$y$
0			1
1	$0+1=1$	$1 \cdot 1$	2
2	$1+2=3$	$3 \cdot 1$	5
3	$2+5=7$	$7 \cdot 1$	12

Exactly

Solution:  $y = 2e^x - x - 1$

Check:

$$\frac{dy}{dx} = 2e^x - 1 = x + (2e^x - x - 1)$$
$$= x + y$$

$$x=0, y = 2e^0 - 0 - 1 = 1$$

$$x=3, y = 2e^3 - 3 - 1 \approx 36.1$$