Math 181 Friday, April 23 Section 9.4

Logistic Equation

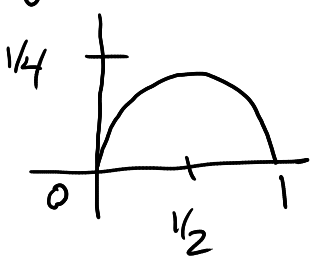
$$\frac{dy}{dt} = ky(A-y)$$

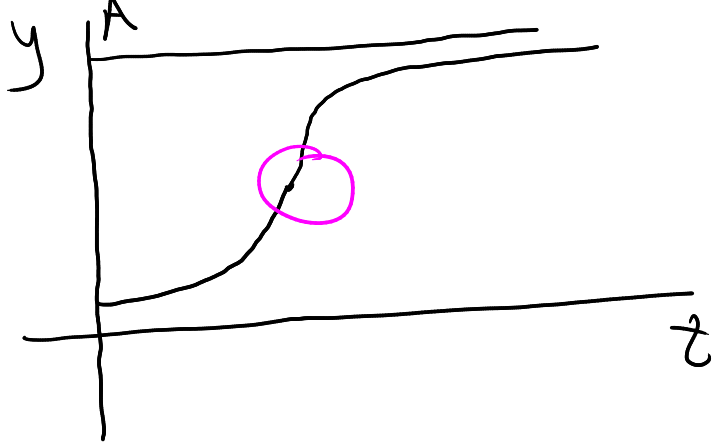
y small $\approx ky(A) = (Ak)y$

exponential growth

y large $\approx ky \cdot 0 = 0$

$y = x(1-x)$





Mon	11.1
Tue	11.2
Thu	
Fri	Problems
Mon	Snow Day
Tue	Exam 8, 9, 11

A – capacity

"no tree grows to the sky"

Solve the Diff EQ

$$\frac{dy}{dt} = Ky(A-y)$$

$$= KyA(1-y/A)$$

$$= (KA)\,y(1-y/A)$$

separable $\Rightarrow$ $= k\,y(1-y/A)$

$$\frac{1}{y(1-y/A)}\,dy = k\,dt$$

Partial Fractions

$$\frac{1}{y} + \frac{\frac{1}{A}}{1-y/A}\,dy = k\,dt$$

$$\frac{1}{y} + \frac{1}{A-y}\,dy = k\,dt$$

$$\frac{1}{y} - \frac{1}{y-A}\,dy = k\,dt$$

$$\int \frac{1}{y} - \frac{1}{y-A}\,dy = \int k\,dt$$

$$\ln y - \ln(y-A) = kt + C$$

$$\ln\left(\frac{y}{y-A}\right) = kt + C$$

$$\frac{y}{y-A} = e^{kt+C} = e^{C}e^{kt} = Be^{kt}$$

$$y = (y-A)\,Be^{kt} = Be^{kt}y - ABe^{kt}$$

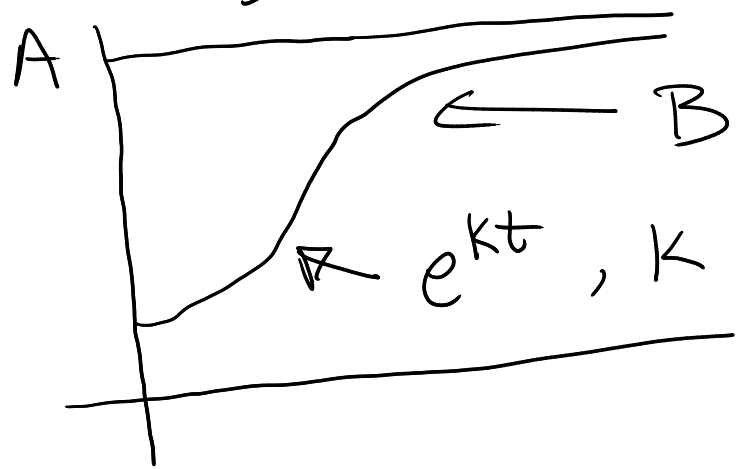
$$y - Be^{kt}y = -ABe^{kt}$$

$$(1-Be^{kt})y = -ABe^{kt}$$

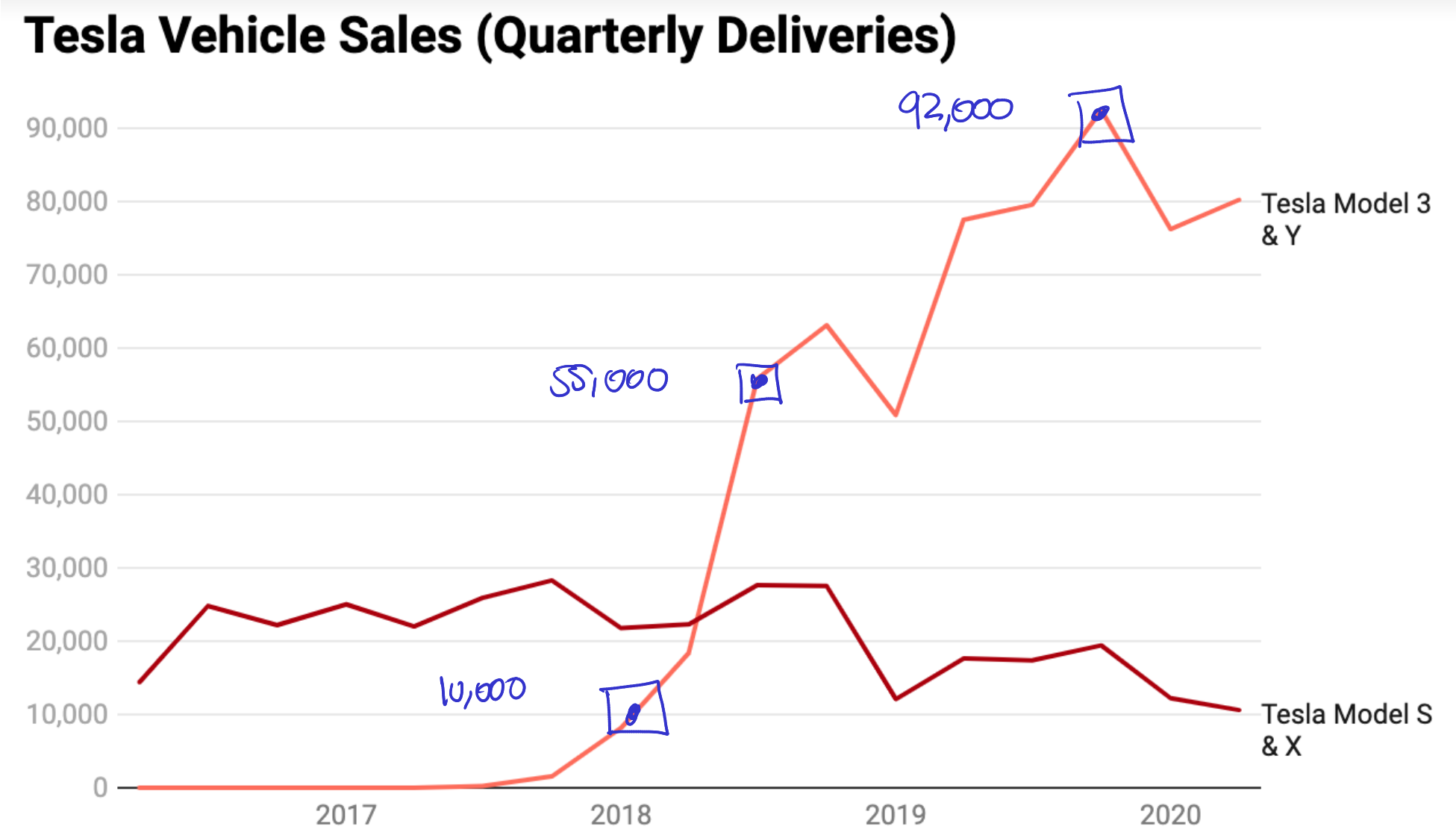
$$y = \frac{-ABe^{kt}}{1-Be^{kt}} \cdot \frac{1/Be^{kt}}{1/Be^{kt}} = \frac{-A}{1/Be^{kt}-1}$$

$$= \frac{-A}{e^{\frac{-Kt}{B}} - 1} = \frac{A}{1 - e^{-Kt}/B}$$

3 parameters K, A, B



Ex Sales of Tesla automobiles



① Estimate k as if growth exponential

$y = 10{,}000\, e^{kt}$ $\quad t=0$ is 2018

$55{,}000 = 10{,}000\, e^{k(1/2)}$ at time 2018.5

$5.5 = e^{k(1/2)}$

$\ln(5.5) = \frac{1}{2}k \rightarrow k = 2\ln(5.5) = 3.40$

Now use logistic equation

2018.5 $\quad 55000 = \dfrac{A}{1-e^{-3.40(0.5)/B}}$

2019.5 $\quad 92{,}000 = \dfrac{A}{1-e^{-3.40(1.5)/B}}$

Sage

$A = 94{,}187.625$

$B = -0.256$