Math 290  Monday, February 22

Section MO (plus MVP)

Tue - Exam V

Laptop! Envelope

Thu - MM

Fri - MIS2E (Sage)

BYOB Movie

Ex. \[ 2 \begin{bmatrix} 3 & 4 \\ -2 & 4 & 0 \end{bmatrix} + 4 \begin{bmatrix} -3 & 2 & 0 \\ 1 & 1 & 6 \end{bmatrix} \]

\[ = \begin{bmatrix} 2 & 6 & 8 \\ -4 & 8 & 0 \end{bmatrix} + \begin{bmatrix} -12 & 8 & 0 \\ 4 & 4 & 24 \end{bmatrix} \]

\[ = \begin{bmatrix} -10 & 14 & 8 \\ 0 & 12 & 24 \end{bmatrix} \]

Other Operations

Transpose

\[ \begin{bmatrix} 2 & 3 & 1 \\ -6 & 4 & 5 \end{bmatrix}^T = \begin{bmatrix} 2 & -6 \\ 3 & 4 \\ 1 & 5 \end{bmatrix} \]

2x3

3x2

Define

\[ [A^T]_{ij} = [A]_{ji} \]
Complex conjugate of a matrix $A$: $\overline{[A]}_{ij} = \overline{[A]}_{ij}$

Adjoint $A^* = (A)^t$ (sometimes "conjugate")

Theorem $(A+B)^t = A^t + B^t$, $A, B$ $m \times n$

Proof For $1 \leq i \leq n$, $1 \leq j \leq m$

$\overline{[(A+B)^t]}_{ij} = \overline{[A+B]}_{ji}$ — Then by Schur-ME

$\overline{[A+B]}_{ji} = \overline{[A]}_{ji} + \overline{[B]}_{ji}$ —

$\overline{[A]}_{ji} = \overline{[A^t]}_{ij}$ —

$\overline{[B]}_{ji} = \overline{[B^t]}_{ij}$ —

$\overline{[A^t + B^t]}_{ij}$ —

Scalar equality

Matrix equality
Section MM

Matrix-Vector Product

Example

\[
\begin{bmatrix}
2 & -1 & 3 \\
4 & 0 & 2
\end{bmatrix}
\begin{bmatrix}
2 \\
-3 \\
1
\end{bmatrix}
= 2 \begin{bmatrix}
2 \\
4
\end{bmatrix} + (-3) \begin{bmatrix}
-1 \\
0
\end{bmatrix} + 1 \begin{bmatrix}
3 \\
2
\end{bmatrix}
\]

Linear combination of columns of the matrix, scalars from vector

\[
\begin{bmatrix}
10 \\
10 \\
20
\end{bmatrix}
\]

System of equations

\[
\begin{align*}
2x_1 - x_2 + 4x_3 &= 6 \\ 9x_1 - 2x_2 + x_3 &= 12
\end{align*}
\]

Matrix equation

\[
\begin{bmatrix}
2 & -1 & 4 \\
9 & 2 & 1
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}
= \begin{bmatrix}
6 \\
12
\end{bmatrix}
\]

Coefficient matrix

\[
A \cdot x = b \parallel LS(\mathbf{A}, \mathbf{b})
\]