Math 290  Monday, March 1

Theorem NP NF: \[ AB \text{ nonsingular } \iff \text{A and B both nonsingular} \]

2nd Grade:
\[ xy = 0 \iff x = 0 \text{ or } y = 0 \]
\[ xy \neq 0 \iff x \neq 0 \text{ and } y \neq 0 \]

\[ \text{non zero } \iff \text{non singular} \]
\[ \text{zero } \equiv \text{ singular} \]

"singular is to matrices as zero is to numbers"

Section M1NM

Tuesday - Problems
Thu - CRS \int \sin x^2
Fri - FS
BYOB - solo outdoor sports

Mon - Problems
Writing M
Tue - Exam M
Proof \((\Rightarrow)\) \(AB\) nonsingular \(\Rightarrow\) \(A\) nonsingular \& \(B\) nonsingular

Contrapositive: \(A\) singular or \(B\) singular \(\Rightarrow\) \(AB\) singular

Case 1: \(B\) singular so there is \(\xi \neq 0\) so that \(B\xi = 0\).

Then \((AB)\xi = A(B\xi) = AQ = 0\), so \(AB\) is singular.

Case 2: \(B\) nonsingular. Then \(A\) is singular. (hypothesis)

There exists \(y \neq 0\) so that \(Ay = 0\).

Solve \(Ls(B, y)\). Unique solution (NMUS), call \(w\)

\[Bw = y.\]

Claim \(w \neq 0\). Suppose \(w = 0\). Then \(y = Bw = B0 = 0\).

Now \((AB)w = A(Bw) = Ay = 0\). So \(AB\) is singular.
\((\leftarrow)\) \(A \& B\) nonsingular \(\Rightarrow\) \(AB\) nonsingular

Consider \(LS(AB,\mathbb{Q})\) solve

\[(AB)\mathbf{x} = \mathbf{0}\]

A nonsingular

\[A(B\mathbf{x}) = \mathbf{0}\]

\(B\mathbf{x} = 0 \Rightarrow \mathbf{x} = \mathbf{0}\)

\[\therefore AB\text{ is nonsingular.}\]

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**Theorem**

\(AB = I \Rightarrow BA = I\)

**Proof**

\[I = AB \Rightarrow B\text{ nonsingular}\]

\(NRRFF\) (\(A\) nonsingular)

Then \(BA = (BA)I = (BA)(BC) = B(AB)C = BIC = BC = I\)

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**Theorem**

\(CINM \Rightarrow C\) so that \(BC = I\)
Theorem NI  A nonsingular $\iff$ A invertible

$(\Leftarrow)$ A invertible, $A^{-1}$ exists \quad $I_n = AA^{-1}$

$(\Rightarrow)$ A nonsingular $\Rightarrow$ A nonsingular

\[ AB = I \Rightarrow BA = I \Rightarrow B = A^{-1} \]

Unitary Matrices

$U^{-1} = U^*$

Theorem UMPUP

$\langle U u, U v \rangle = \langle u, v \rangle$

$U \cdot v = ||u|| ||v|| \cos \theta \quad ||Uv|| = ||v||$