Math 290 Thursday, March 4

Section CRS

RQ CSNM v. NMEV

Fri - FS
BYOB Solo Outdoor Sport

Mon - Problems
Writing M

Tue - Exam M

Column Space $C(A)$

$A = [A_1 | A_2 | \cdots | A_n]$

$C(A) = \langle \{A_1, A_2, \ldots, A_n\} \rangle$

Theorem CSCS

$b \in C(A) \iff Ax = b$ consistent

$Ax = b$

$[A_1 | A_2 | \cdots | A_n]\, x = b$

$[x]_1 A_1 + [x]_2 A_2 + \cdots + [x]_n A_n = b$ \ SLSLC

"Column space of A is all b vectors where $Ax = b$ is consistent"

Ex

$$A = \begin{bmatrix} 5 & 10 & -1 & 1 & 18 \\ -4 & -8 & 4 & 3 & -10 \\ 1 & 2 & -1 & -1 & 2 \\ -3 & -6 & 2 & 1 & -9 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} \textcircled{1} & 2 & 0 & 0 & 3 \\ 0 & 0 & \textcircled{1} & 0 & -1 \\ 0 & 0 & 0 & \textcircled{1} & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = B$$

$$D = \{1, 3, 4\}$$

$C(A) = ?$

= span of columns — Theorem BS

row reduce, identify pivot columns

use columns 1, 3 & 4

$$= \left\langle \left\{ \begin{bmatrix} 5 \\ -4 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ -1 \\ 1 \end{bmatrix} \right\} \right\rangle$$

linearly independent set

Theorem BCS

Solve $\underset{\sim}{A x} = \begin{bmatrix} 9 \\ 2 \\ -3 \\ -2 \end{bmatrix} = \underset{\sim}{b}$ $\quad [A \mid \underset{\sim}{b}] \xrightarrow{RREF} \left[B \,\middle|\, \begin{matrix} -2 \\ -9 \\ 10 \\ \boxed{0} \end{matrix} \right]$ system is consistent $\underset{\sim}{b} \in C(A)$

So $\underset{\sim}{b} = a_1 \begin{bmatrix} 5 \\ -4 \\ 1 \\ -3 \end{bmatrix} + a_2 \begin{bmatrix} -1 \\ 4 \\ -1 \\ 2 \end{bmatrix} + a_3 \begin{bmatrix} 1 \\ 3 \\ -1 \\ 1 \end{bmatrix}$ has a solution

make a system (SLSLC, augmented matrix ...)

$$\left[\begin{array}{ccc|c} 5 & -1 & 1 & 9 \\ -4 & 4 & 3 & 2 \\ 1 & -1 & -1 & -3 \\ 3 & 2 & 1 & -2 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{ccc|c} \textcircled{1} & 0 & 0 & -2 \\ 0 & \textcircled{1} & 0 & -9 \\ 0 & 0 & \textcircled{1} & 10 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$a_1 = -2$
$a_2 = -9$
$a_3 = 10$

(solution to $\underset{\sim}{A x} = \underset{\sim}{b}$ w/ free variables equal to zero)

Solve $\underset{\sim}{A x} = \begin{bmatrix} 3 \\ 3 \\ -1 \\ 4 \end{bmatrix} = \underset{\sim}{c}$ $\quad [A \mid \underset{\sim}{c}] \xrightarrow{RREF} \left[B \,\middle|\, \begin{matrix} 0 \\ 0 \\ 0 \\ \textcircled{1} \end{matrix} \right]$ no solution, inconsistent, $\underset{\sim}{c} \notin C(A)$

Row Space

$$R(A) = C(A^t)$$

Theorem REMRS Row-equivalent matrices have equal row spaces.

Theorem BRS The nonzero rows of RREF span the row space and are linearly independent

"Probably the most powerful computational technique at your disposal"

Ex $A_{4\times 6}$ $\quad C(A) = ?$

$$C(A) = R(A^t)$$

$$A^t \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} -1/4 \\ 1/2 \\ 1/4 \end{matrix}$$

$$C(A) = \left\langle \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\} \right\rangle$$

$$\mathbf{b} = \begin{bmatrix} 9 \\ 2 \\ -3 \\ -2 \end{bmatrix} = 9\begin{bmatrix} 1 \\ 0 \\ 0 \\ -1/4 \end{bmatrix} + 2\begin{bmatrix} 0 \\ 1 \\ 0 \\ 1/2 \end{bmatrix} + (-3)\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1/4 \end{bmatrix}$$

↑ checks