Math 290

Thursday, March 25

~~Determinants~~

~~Characteristic Polynomial~~

Section EE

· Writing / afternoon

Fri - PEE

Break let

Thu - IS

Fri - Problems

<u>Defn</u> A square matrix, with

$$A\underset{\sim}{x} = \lambda \underset{\sim}{x} \quad , \quad \underset{\sim}{x} \neq \underset{\sim}{0}$$

$\underset{\sim}{x}$ is an <u>eigenvector</u> of A, with <u>eigenvalue</u> λ.

Q: What are the eigenvalues of A?

Theorem ESM

λ is an eigenvalue of $A \iff A - \lambda I$ is a singular matrix

Proof $\quad A\underset{\sim}{x} = \lambda \underset{\sim}{x} \quad , \underset{\sim}{x} \neq \underset{\sim}{0}$

$$A\underset{\sim}{x} - \lambda \underset{\sim}{x} = \underset{\sim}{0}$$

$$A\underset{\sim}{x} - \lambda I \underset{\sim}{x} = \underset{\sim}{0}$$

$$(A - \lambda I)\underset{\sim}{x} = \underset{\sim}{0} \iff \underset{\sim}{x} \in \underbrace{N(A-\lambda I)}, \ \underset{\sim}{x} \neq \underset{\sim}{0}$$

eigenspace of λ, populated w/ eigenvectors (except $\underset{\sim}{0}$)

$\mathcal{E}_A(\lambda)$

Defn The geometric multiplicity of λ is the dimension of $\mathcal{E}_A(\lambda)$.

$$\gamma_A(\lambda) = \dim(\mathcal{E}_A(\lambda))$$

Ex $A = \begin{bmatrix} 8 & -6 & 6 \\ 6 & -4 & 6 \\ -3 & 3 & -1 \end{bmatrix}$ Grab any $\underset{\sim}{x} \neq \underset{\sim}{0}$, $\underset{\sim}{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \underset{\sim}{e_1}$

Build $A^0 \underset{\sim}{e_1}$, $A^1 \underset{\sim}{e_1}$, $A^2 \underset{\sim}{e_1}$, $A^3 \underset{\sim}{e_1}$ 4 column vectors in $\mathbb{C}^3$

$\Rightarrow$ linearly dependent set

$$[A^0\underset{\sim}{e_1} | A^1\underset{\sim}{e_1} | A^2\underset{\sim}{e_1} | A^3\underset{\sim}{e_1}] = \begin{bmatrix} 1 & 8 & 10 & -10 \\ 0 & 6 & 6 & -18 \\ 0 & -3 & -9 & -21 \end{bmatrix} \xrightarrow[\text{RREF}]{} \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -8 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$x_4 = \text{free} = 1$

$\begin{bmatrix} -4 \\ 8 \\ -5 \\ 1 \end{bmatrix}$ relation of linear dependence

$$-4A^0\underset{\sim}{e_1} + 8A^1\underset{\sim}{e_1} - 5A^2\underset{\sim}{e_1} + 1A^3\underset{\sim}{e_1} = \underset{\sim}{0}$$

$$(-4I + 8A - 5A^2 + A^3)\underset{\sim}{e_1} = \underset{\sim}{0}$$

$$(A-I)(A-2I)(A-2I)\underset{\sim}{e_1} = 0 \leftarrow p(x) = -4 + 8x - 5x^2 + x^3$$

$$= \boxed{(x-1)}(x-2)^2$$

$$(A-2I)\underbrace{(A-I)(A-2I)\underset{\sim}{e_1}}_{\underset{\sim}{y}} = 0$$

$(A-2I)\underset{\sim}{y} = \underset{\sim}{0} \rightarrow A\underset{\sim}{y} - 2\underset{\sim}{y} = \underset{\sim}{0}$

$A\underset{\sim}{y} = 2\underset{\sim}{y}$ 2 is an eigenvalue of A

EMHE

Eigenspaces: $\lambda = 2$ root of $p(x)$, $\lambda = 2$ <u>may</u> be an eigenvalue

$$A - 2I = \begin{bmatrix} 6 & -6 & 6 \\ 6 & -6 & 6 \\ -3 & 3 & -3 \end{bmatrix}$$

SINGULAR

<u>now</u> we know 2 <u>is</u> an eigenvalue

$$\xrightarrow{\text{RREF}} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathcal{E}_A(2) = N(A - 2I)$$

$$= \left\langle \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\} \right\rangle$$

$$\gamma_A(2) = 2$$

- Companion Matrices
- Example UE