Math 290   Friday, April 2

Problem Session
Mon- SD (Sage)
Tue- Exam VS
Thu- ME

IS. T30

NPSM

$$N(A^k) \subseteq N(A^{k+1})$$

Prove $C(A^k) \supseteq C(A^{k+1})$

Grab $x \in C(A^{k+1})$ then there exists y so that $A^{k+1}y = x$

MVP = linear combination

$$x = A^{k+1}y = A^k A y = A^k(w)$$

vector

MVP = linear combination of columns of A^k

So $x \in C(A^k)$

EE. C27

IS. T20

$$G_A(\lambda) = \{ \underset{\sim}{x} \mid (A - \lambda I)^k \underset{\sim}{x} = \underset{\sim}{0} \text{ for some } k \geq 0 \}$$

subspace of $\mathbb{C}^n$? Use TSS

1) $\underset{\sim}{0} \in G_A(\lambda)$. (true w/ $k=0$, $k=1$ or ...) $\Rightarrow G_A(\lambda)$ non-empty

2) If $\underset{\sim}{x} \in G_A(\lambda)$ is $\alpha \underset{\sim}{x} \in G_A(\lambda)$?

Know $(A - \lambda I)^{\hat{k}} \underset{\sim}{x} = \underset{\sim}{0}$.

Then $(A - \lambda I)^{\hat{k}} (\alpha \underset{\sim}{x}) = \alpha (A - \lambda I)^{\hat{k}} \underset{\sim}{x} = \alpha \cdot \underset{\sim}{0} = \underset{\sim}{0}$

So $\alpha \underset{\sim}{x} \in G_A(\lambda)$.

3) Grab $\underset{\sim}{x} \in G_A(\lambda)$, $\underset{\sim}{y} \in G_A(\lambda)$. Is $\underset{\sim}{x} + \underset{\sim}{y} \in G_A(\lambda)$?

Know $\underbrace{(A-\lambda I)^{\hat{k}}\, \underset{\sim}{x}} = \underset{\sim}{0}$ & $\underbrace{(A-\lambda I)^{k^*}\, \underset{\sim}{y} = \underset{\sim}{0}}$

Then $(A-\lambda I)^{\hat{k}+k^*}(\underset{\sim}{x}+\underset{\sim}{y}) = (A-\lambda I)^{\hat{k}+k^*}\underset{\sim}{x} + (A-\lambda I)^{\hat{k}+k^*}\underset{\sim}{y}$

$$= (A-\lambda I)^{k^*}\underbrace{(A-\lambda I)^{\hat{k}}\underset{\sim}{x}} + (A-\lambda I)^{\hat{k}}\underbrace{(A-\lambda I)^{k^*}\underset{\sim}{y}}$$

$$= (A-\lambda I)^{k^*}\underset{\sim}{0} + (A-\lambda I)^{\hat{k}}\underset{\sim}{0} = \underset{\sim}{0} + \underset{\sim}{0} = \underset{\sim}{0}$$

So $\underset{\sim}{x} + \underset{\sim}{y} \in G_A(\lambda)$