Math 390 Thursday, January 21 Vector Space Properties

Friday - BYOB Comics
- Linear Transformations

Subspaces

P_3 = vector space of polynomials degree at most 3

$$W = \{ a+bx+cx^2+dx^3 \mid a+b-5c-3d=0,\ 3a+4b-17c-11d=0 \}$$

Fact W is a subspace

linear combination

span $\langle \{ \underset{\sim}{x}_1, \underset{\sim}{x}_2, \dots, \underset{\sim}{x}_m \} \rangle = \{ a_1 \underset{\sim}{x}_1 + a_2 \underset{\sim}{x}_2 + \cdots + a_m \underset{\sim}{x}_m \mid a_i \in F \}$

subspace

S **spans** W **or** S is a **spanning set** for W

if $W = \langle S \rangle$

Spanning set for W

$$a+b-5c-3d=0$$
$$3a+4b-17c-11d=0$$

$$A\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad A \xrightarrow{RREF} \begin{bmatrix} 1 & 0 & -3 & -1 \\ 0 & 1 & -2 & -2 \end{bmatrix}$$

$$W = \{ a+bx+cx^2+dx^3 \mid a-3c-d=0,\ b-2c-2d=0 \}$$
$$= \{ a+bx+cx^2+dx^3 \mid a=3c+d,\ b=2c+2d \}$$
$$= \{ (3c+d)+(2c+2d)x+cx^2+dx^3 \mid c,d \in \mathbb{C} \}$$
$$= \{ c(3+2x+x^2)+d(1+2x+x^3) \mid c,d \in \mathbb{C} \}$$
$$= \langle \underbrace{\{ 3+2x+x^2,\ 1+2x+x^3 \}}_{\text{spanning set for } W} \rangle$$

Linear Independence Relation of Linear Dependence

$S = \{\underset{\sim}{x_1}, \dots, \underset{\sim}{x_m}\}$ $a_1\underset{\sim}{x_1} + a_2\underset{\sim}{x_2} + \dots + a_m x_m = \underset{\sim}{0} \Rightarrow a_1 = a_2 = \dots = a_m = 0$

$T = \{3+2x+x^2, 1+2x+x^3\}$ is linearly independent

$a_1(3+2x+x^2) + a_2(1+2x+x^3) = \underset{\sim}{0}$

$(3a_1+a_2) + (2a_1+2a_2)x + a_1x^2 + a_2x^3 = 0+0x+0x^2+0x^3$

$\Rightarrow a_1 = 0, \; a_2 = 0$

T is a basis of W (linearly independent spanning set) Theorem G

So $\dim(W) = 2$. (Isomorphic to $\mathbb{C}^2$)

$R = \{4-4x+3x^2-5x^3, 6+4x+2x^2, -9-2x-4x^2+3x^3\} \subseteq W$ $\boxed{\text{linearly dependent} \; 3>2}$

$K = \{6+4x+2x^2\} \subseteq W$ $\boxed{\text{does not span } W \quad 1<2}$

$L = \{4-4x+3x^2-5x^3, 6+4x+2x^2\}$ show linearly independent $\Rightarrow$ $\boxed{2=2 \text{ so spans}}$

Theorem BIS : Any two bases have identical sizes.

Theorem SSLD

set of size t spans W $\Rightarrow$ any set of $t+1$ or more vectors is linearly dependent

Theorem G

Proof turns on SSLD in all four parts.

(Theorem VRRB , Orthogonality)