Math 390 Friday January 29 FCLA PEE +/-

Theorem EDELI

$S = \{\underset{\sim}{x}_1, \underset{\sim}{x}_2, \dots, \underset{\sim}{x}_p\}$ eigenvectors of A for <u>distinct</u> eigenvalues.

$\Rightarrow$ S is linearly independent

Theorem MNEM A $n \times n$ matrix, A has no more than n distinct eigenvalues.

Theorem SMZE A singular $\Longleftrightarrow$ A has a zero eigenvalue

A nonsingular $\Longleftrightarrow$ A has no zero eigenvalues

<u>Proof</u> 0 eigenvalue $\Longleftrightarrow$ $A - 0I_n$ singular

(λ eigenvalue $\Longleftrightarrow$ $A - \lambda I_n$ singular) theorem EOMP λ eigenvalue of $A \Rightarrow \lambda^k$ eigenvalue of A^k

<u>Proof</u> $\underset{\sim}{x}$ eigenvector of A for λ. Then

$$A^k \underset{\sim}{x} = A^{k-1} A \underset{\sim}{x} = A^{k-1}(\lambda \underset{\sim}{x})$$

$$= \lambda (A^{k-1} \underset{\sim}{x}) = \lambda (A^{k-2} A \underset{\sim}{x}) = \lambda A^{k-2}(\lambda \underset{\sim}{x})$$

$$= \lambda \lambda A^{k-2} \underset{\sim}{x} = \lambda^2 A^{k-2} \underset{\sim}{x}$$

$$\vdots$$

$$= \lambda^l A^{k-l} \underset{\sim}{x} = \cdots = \lambda^k \underset{\sim}{x}$$

$\underset{\sim}{x}$ is an eigenvector of A^k for (with eigenvalue λ^k.

theorem EPM λ eigenvalue of $A \Rightarrow p(\lambda)$ eigenvalue of $p(A)$

A

$p(x)$ polynomial

Hermitian matrices $A^* = \bar{A}^t = A$ "symmetric"

adjoint

Theorem A Hermitian, λ eigenvalue $\Rightarrow \lambda \in \mathbb{R}$

Theorem $\underset{\sim}{x}, \underset{\sim}{y}$ eigenvectors of Hermitian matrix A for different eigenvalues $\Rightarrow \underset{\sim}{x}$ & $\underset{\sim}{y}$ orthogonal