Math 390 Monday, February 1 FCLA IS

Friday - BYOB
Beach scene

Tue - Problems
- Add Generalized Eigenspaces

Thu - SD

PEE - eigenvalues of A^t
= eigenvalues of A

Proof EZ: w/ determinant
Hard: w/o determinant

Defn A - square matrix size n
V - subspace of $\mathbb{C}^n$

V is A-invariant if whenever

$$\underset{\sim}{x} \in V \Rightarrow A\underset{\sim}{x} \in V$$

Good for defining new linear transformations, "restrictions"

$$T : \mathbb{C}^n \to \mathbb{C}^n \quad T(\underset{\sim}{x}) = A\underset{\sim}{x}$$

$$T|_V : V \to V \quad T(\underset{\sim}{x}) = A\underset{\sim}{x}$$

Example TIS

Theorem NSPIS

A - square, $p(x)$ - polynomial $\Rightarrow$ $N(p(A))$ an A-invariant subspace

Corollary
1. $N(A)$ is A-invariant
2. $N(A^k)$ is A-invariant
3. $N(\underbrace{A-\lambda I}_{p(x)=x-\lambda})$ is A-invariant $\leftarrow$ eigenspace

Eigenspaces are A-invariant

$\underset{\sim}{x} \in \mathcal{E}_A(\lambda)$ then

$A\underset{\sim}{x} = \lambda \underset{\sim}{x} \in \mathcal{E}_A(\lambda)$

$\uparrow$ scalar closure

Proof Grab $\underset{\sim}{z} \in N(p(A))$ [Know $p(A)\underset{\sim}{z} = \underset{\sim}{0}$]

[Then is $A\underset{\sim}{z} \overset{?}{\in} N(p(A))$]

$p(A)(A\underset{\sim}{z}) = A\,p(A)\underset{\sim}{z} = A\,\underset{\sim}{0} = \underset{\sim}{0}$, so yes

Theorem NSPM $\quad A$ square of size n.

$$\{0\} = N(A^0) \subsetneq N(A) \subsetneq N(A^2) \subsetneq N(A^3) \subsetneq \cdots \subsetneq N(A^m) = N(A^{m+1}) = N(A^{m+2}) = \cdots$$

A "topping out"

$m =$ index

<u>Proof</u> $\quad N(A^k) \subseteq N(A^{k+1})$

Grab $\underset{\sim}{x} \in N(A^k)$, examine $A^{k+1}\underset{\sim}{x} = A(A^k\underset{\sim}{x}) = A\underset{\sim}{0} = \underset{\sim}{0}$

So $\underset{\sim}{x} \in N(A^{k+1})$.

<u>FACT</u> $\quad m \leq n$

<u>Generalized Eigenspaces</u> $\quad$ suppose λ eigenvalue of A of size n.

$$G_A(\lambda) = N((A - \lambda I)^n)$$