Math 390 Thursday, February 4 Similarity FCLA SD

A similar to B $\iff$ there exist nonsingular S so that $S^{-1}AS = B$

Equivalence Relation: $A \sim A$ reflexive

$A \sim B \Rightarrow B \sim A$ symmetric

$A \sim B,\ B \sim C \Rightarrow A \sim C$ transitive

$$\underbrace{S^{-1}AS}_{\text{"similarity transformation"}} = B \iff \boxed{AS = BS}$$

Theorem PSMS

$$AS = SB \implies p(A)\,S = S\,p(B)$$

<u>Proof</u>

$$\begin{aligned} A^k S &= A^{k-1} AS = A^{k-1} SB \\ &= A^{k-2} \underline{AS}B = A^{k-2} \underline{SB}\,B = A^{k-2} SB^2 \\ &\;\vdots \\ &= A^{k-i} SB^i \\ &\;\vdots \\ &= SB^k \end{aligned}$$

Theorem SMEE

A & B similar $\Rightarrow$ A & B have the same eigenvalues, geometric multiplicity, algebraic multiplicity

Proof $AS = SB$

Claim $\dim(N((B-\lambda I)^k)) \leq \dim(N((A-\lambda I)^k))$

so $=$

basis of $\{\underset{\sim}{x}_1, \underset{\sim}{x}_2, \ldots, \underset{\sim}{x}_\ell\}$ $\{S\underset{\sim}{x}_1, S\underset{\sim}{x}_2, \ldots, S\underset{\sim}{x}_\ell\}$

linearly independent set MM.T80

Look at

$$(A-\lambda I)^k S\underset{\sim}{x}_i = S\underbrace{(B-\lambda I)^k \underset{\sim}{x}_i}$$

PSMS

$$= S\underset{\sim}{0}$$

$$= \underset{\sim}{0}$$ So $S\underset{\sim}{x}_i$ is in

$$\cancel{S^{-1}AS = B} \qquad AS = SB$$

$$A\,[\underset{\sim}{x}_1 \mid \underset{\sim}{x}_2 \mid \cdots \mid \underset{\sim}{x}_n] = [\underset{\sim}{x}_1 \mid \underset{\sim}{x}_2 \mid \cdots \mid \underset{\sim}{x}_n] \begin{bmatrix} \lambda_1 & 0 & & 0 \\ 0 & \lambda_2 & & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & & \lambda_n \end{bmatrix}$$

eigenvectors

$$[A\underset{\sim}{x}_1 \mid A\underset{\sim}{x}_2 \mid \cdots \mid A\underset{\sim}{x}_n]$$

$$A\underset{\sim}{x}_1 = \lambda_1 \underset{\sim}{x}_1$$

$$A\underset{\sim}{x}_2 = \lambda_2 \underset{\sim}{x}_2$$

$$\vdots$$

$$A\underset{\sim}{x}_n = \lambda_n \underset{\sim}{x}_n$$