Math 3010 Friday, February 5 Similarity 2 FCLA SD

Theorem DC

Mat CP

The Problems

A diagonalizable $\iff$ there is a basis for $\mathbb{C}^n$ where every vector is an eigenvector of A

Proof ($\Leftarrow$) $\{\underset{\sim}{x}_1, \underset{\sim}{x}_2, \ldots, \underset{\sim}{x}_n\}$ basis of eigenvectors

$$AS = A[\underset{\sim}{x}_1|\underset{\sim}{x}_2|\cdots|\underset{\sim}{x}_n]$$
$$= [A\underset{\sim}{x}_1|A\underset{\sim}{x}_2|\cdots|A\underset{\sim}{x}_n]$$
$$= [\lambda_1\underset{\sim}{x}_1|\lambda_2\underset{\sim}{x}_2|\cdots|\lambda_n\underset{\sim}{x}_n] \longleftrightarrow = [\lambda_1\underset{\sim}{x}_1|\lambda_2\underset{\sim}{x}_2|\cdots|\lambda_n\underset{\sim}{x}_n]$$

$$SD = [\underset{\sim}{x}_1|\underset{\sim}{x}_2|\cdots|\underset{\sim}{x}_n]\begin{bmatrix}\lambda_1 & & 0\\ & \lambda_2 & \\ & & \ddots & \\ 0 & & & \lambda_n\end{bmatrix}$$

Theorem SUT

A square $\Rightarrow$ A similar to an upper-triangular matrix

Proof

Claim U is an A-invariant subspace

$\Rightarrow$ basis $B = \{\underset{\sim}{x}_1, \underset{\sim}{x}_2, \ldots, \underset{\sim}{x}_n\}$ with

$$A x_k = \hat{x} + \underline{\lambda_k \underset{\sim}{x}_k}$$

① $\lambda_k =$ eigenvalue of A

② $\underset{\sim}{x} \in \underline{\langle\{\underset{\sim}{x}_1, \ldots, \underset{\sim}{x}_{k-1}\}\rangle}$

$$AS = S \begin{bmatrix} & x & \\ & x & \\ & x & \\ & x & \\ & \lambda_k & \\ & 0 & \\ & 0 & \end{bmatrix}$$