Math 390 Thursday, February 11

Matrices (290) $\longrightarrow$ Linear Transformations (390)

Section MR

$$(S \circ T)(\underset{\sim}{u}) = S(T(\underset{\sim}{u}))$$

<u>MRCLT</u>

$T: U \to V \quad S: V \to W$

bases $\quad B \quad C \qquad C \quad D$

$$M^{S \circ T}_{B,D} = M^{S}_{C,D} \, M^{T}_{B,C}$$

matrix multiplication

Fri - SCLA Direct Sums 1.2
Orthogonal Complements 1.3
(Not on Exam)

Exam - Sage

Mon - Review / Problems
MR.T80

Tue - Exam

<u>IMR</u>

$$M^{T^{-1}}_{C,B} = \left(M^{T}_{B,C}\right)^{-1}$$

SCLA: Matrix A $n \times n$ $\Longleftrightarrow$ $T: \mathbb{C}^n \to \mathbb{C}^n \quad T(\underset{\sim}{x}) = A\underset{\sim}{x}$

Section CB

Change-of-Basis V - vector space, two bases B, C

$$C_{B,C} = M^{I_V}_{B,C}$$ identity on V $(I_V : V \to V)$

<u>CB</u> $\rho_C(\underset{\sim}{v}) = C_{B,C} \, \rho_B(\underset{\sim}{v})$ $\quad (C_{C,B} = C^{-1}_{B,C})$

column vectors

matrix-vector product

<u>MRCB</u> $T : \underset{B,C}{U} \to \underset{D,E}{V}$

$M^T_{B,D} \, \rho_B(\underset{\sim}{u}) = \rho_D(T(\underset{\sim}{u}))$

$\rho_D(T(\underset{\sim}{u})) \leftarrow \rho_E(T(\underset{\sim}{u})) \leftarrow \rho_C(\underset{\sim}{u}) \leftarrow \rho_B(\underset{\sim}{u})$

$$M^T_{B,D} = C_{E,D} \, M^T_{C,E} \, C_{B,C}$$

Specialize to $U = V$, $D = B$, $E = C$

$$M^T_{B,B} = C_{C,B} \, M^T_{C,C} \, C_{B,C} = C^{-1}_{B,C} \, M^T_{C,C} \, C_{B,C}$$

SIMILARITY

A is similar to B , $AS = SB$

$$\boxed{\text{SAME AS}}$$

$T(\underset{\sim}{x}) = A\underset{\sim}{x}$ find a basis of $\mathbb{C}^n$, C, so

$$M^T_{C,C} = B$$

"Finding S-matrix of similarity is finding a basis to provide a matrix representation"

Eigenvalues of a linear transformation

$T : V \to V$ $\qquad$ $T(\underset{\frown}{x}) = \lambda \underset{\frown}{x}$

eigenvector

eigenvalue

<u>Theorem</u> Similar matrices have
SMEE equal eigenvalues.

"Eigenvalues of a linear transformation are independent of matrix representation."

$$N(A) \cong K(T)$$