Math 390  Monday  February 22

The - Regular
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Fam

Fri - BYOB Movies
3.2 Nilpotent Linear Transformations

**Definition:** \( T : V \rightarrow V \) is *nilpotent* with index \( p \) if (square matrices)

\[
T^p(v) = 0 \quad \text{for all } v \in V.
\]

(matrices: \( A^p = 0 \))

\[
T(T(T(...))) = T^p(\cdot)
\]

Then \( T \) nilpotent \( \Rightarrow \) any eigenvalue of \( T \) is \( \lambda = 0 \).

**Proof:** \( \lambda \) eigenvalue of \( T \) for \( \lambda x = \lambda \).

\[
0 = T^p(x) = \lambda^p x, \quad x \neq 0
\]

\[
\Rightarrow \lambda^p = 0 \Rightarrow \lambda = 0
\]

**Theorem:** EPM

Eigenvalues of Polynomial of a Matrix
Theorem 3.2.5

\[ T: V \rightarrow V \text{ nilpotent}. \]

\[ T \text{ diagonalizable } \iff T \text{ is the zero linear transformation.} \]

\[ \left( T(v) = 0 \text{ for all } v \in V \right) \]

\[ \uparrow \text{ basis w/ diagonal representation.} \]
Feb 11: \( A \) is similar to \( B \), \( AS = SB \)

\[ \begin{align*}
    T(x) &= A x \\
    \text{find a basis of } C^n, C, \text{ and } \\
    M_{C, C}^T &= B
    \end{align*} \]

Finding S-matrix of similarity is finding a basis to provide a matrix representation.

Chapter 5 (FCA): Eigenvalues are hard. Everything else is easy.

See worksheet "CP-class"