

Math 390 Thursday, February 25

Jordan Canonical Form

$$J_3(5) = \begin{bmatrix} 5 & 1 & 0 \\ 0 & 5 & 1 \\ 0 & 0 & 5 \end{bmatrix}$$

$$J_3(5) - 5I_3 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

nilpotent

Thu JCF

Fri JCF

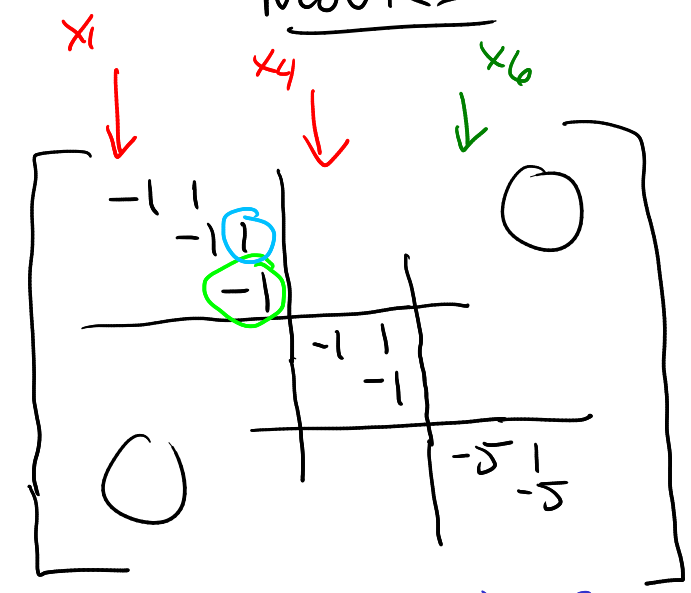
Movies

Defn Jordan Canonical Form

A matrix that

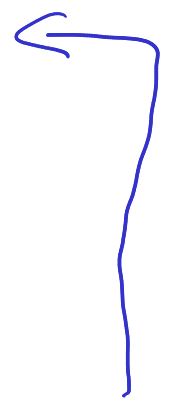
- ① Block diagonal
- ② Each block is Jordan block
- ③ If $\lambda \neq \rho$ eigenvalues w/ $\lambda > \rho$ then $J(\lambda)$ is "higher" than $J(\rho)$
- ④ For each eigenvalue, larger blocks are higher.

Ex



$$\begin{aligned} \alpha(-1) &= 5 \\ \gamma(-1) &= 2 \\ \epsilon(-1) &= 3 \end{aligned}$$

$$\begin{aligned} \alpha(-5) &= 2 \\ \gamma(-5) &= 1 \\ \epsilon(-5) &= 2 \end{aligned}$$



Theorem 3.3.5 Every linear transformation has a basis $T: V \rightarrow V$

that yields a matrix representation in JCF

[Theorem 3.3.1
main argument]

PLUS • λ eigenvalue of $T \iff J_k(\lambda)$ is a block

- largest block for eigenvalue λ has size $\nu(\lambda)$ \leftarrow index
- Number of blocks for an eigenvalue is $\delta(\lambda) \leftarrow$ geom. mult.
- Total of sizes for all blocks is $\alpha(\lambda) \leftarrow$ alg. mult.

Σ

Jordan Chains $T: V \rightarrow V$, λ eigenvalue of index p , $K_i = K((T - \lambda I)^i)$

Jordan Chains

$$\underline{x}_p \in K_p \setminus K_{p-1}$$

$$\underline{x}_{p-1} = (T - \lambda I) \underline{x}_p, \quad \underline{x}_{p-2} = (T - \lambda I) \underline{x}_{p-1}$$

$$\dots \quad \underline{x}_{k1} = (T - \lambda I) \underline{x}_k$$

FACTS

① $\underline{x}_k \in K_k$

$$\begin{aligned}
 (T - \lambda I)^k \underline{x}_k &= (T - \lambda I)^k (T - \lambda I) \underline{x}_{k+1} \\
 &= (T - \lambda I)^{k+1} \underline{x}_{k+1} \\
 &= (T - \lambda I)^{k+1} (T - \lambda I) \underline{x}_{k+2} \\
 &= (T - \lambda I)^{k+2} \underline{x}_{k+2} = \dots = (T - \lambda I)^p \underline{x}_p = \underline{0}
 \end{aligned}$$

↙ $\underline{x}_p \in K_p$

② \underline{x}_1 is a traditional eigenvector
 $\underline{x}_1 \in K_1 = K((T - \lambda I)^1) = K(T - \lambda I) = \mathcal{E}_T(\lambda)$

③ $\underline{x}_{k-1} = (T - \lambda I) \underline{x}_k$
 $= T \underline{x}_k - \lambda \underline{x}_k$

Reverse

$$T \underline{x}_k = \underline{0} \underline{x}_{k-1} + \lambda \underline{x}_k$$

$$(T \underline{x}_1 = \lambda \underline{x}_1)$$

pesky 1 \rightarrow $\underline{1} \underline{x}_{k-1}$ \uparrow almost diagonal

$\underline{x}_0 = \underline{0}$