Math 390 Thursday, March 4

## RREF

$A \xrightarrow{\text{RREF}} B$ "process"

$B = E_m E_{m-1} \cdots E_1 A$

elementary matrices, row operations

$= J A$ ← nonsingular

$\Rightarrow$ $A = J^{-1} B$

$A$: original; $J^{-1}$: nonsingular; $B$: in RREF ($B$ unique)

## LU Decomposition

- LU
- QR
- SVD
- Cholesky
- Schur

] SCLA, Trefethen & Bau (TB)

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$[A | I_m]$ ($m \times n$) $\xrightarrow{\text{RREF}}$ $[\ \ | J]$

"extended echelon form"

# LU Decomposition

## FLA Triangular Matrices

§ M0 Definitions UTM, LTM

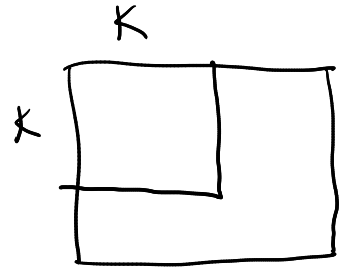
§ MM Theorem PTMT

A, B triangular ⇒ AB triangular

§ MINM A triangular ⇒ $A^{-1}$ triangular

Theorem ITMT

A: square, each kxk matrix in upper-left corner is nonsingular




A row operations to RREF (but not completely)

Don't clear out columns above the diagonal

Get an upper-triangular matrix

$$U = L_m \cdots L_2 L_1 A$$

$U$: upper-triangular

$L_m \cdots L_2 L_1$: type 3 row operations (multiply row by scalar, add to row below.)

$$= LA$$

$L$: lower triangular

Then

$$L^{-1}U = A$$

$L^{-1}$: lower triangular

$$A = PLU$$

$P$: permutation

Solve $A\underset{\sim}{x} = \underset{\sim}{b}$

w/ LU decomposition

Strassen $O(n^{2.7})$

$$LU\underset{\sim}{x} = \underset{\sim}{b}$$

$$L(U\underset{\sim}{x}) = \underset{\sim}{b}$$

solve for $U\underset{\sim}{x} = \underset{\sim}{y}$

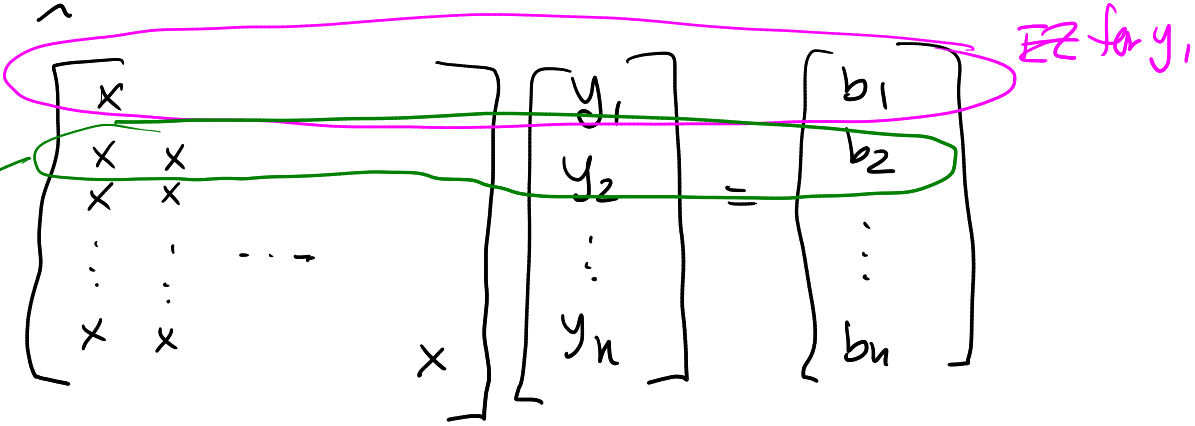
① $L\underset{\sim}{y} = \underset{\sim}{b}$ ←

then

② $U\underset{\sim}{x} = \underset{\sim}{y}$

$2y_1 + 3y_2 = 8$

EZ for $y_2$




forward-solving

$U\underset{\sim}{x} = \underset{\sim}{y}$

back-solving