Math 390 Friday, March 5 LU & QR Decompositions

Strassen Multiplication

$$P = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix}$$

$T = \underline{ae} - dh$

$[P]_{11} = T - Q + R$

$\mathcal{O}(n^3) / \mathcal{O}(n^{2.7\ldots})$

Thm 2.1.1 LU Decomposition is unique

Suppose $A = L_1 U_1 = L_2 U_2$ (L_1, L_2 have 1's on diagonal)

$$\underbrace{\underline{L_2^{-1}} L_1}_{\text{l.t.}} = L_2^{-1} A A^{-1} L_1$$

$$= L_2^{-1} (L_2 U_2)(L_1 U_1)^{-1} L_1$$

$$= L_2^{-1} L_2 U_2 U_1^{-1} L_1^{-1} L_1$$

$$= \underbrace{U_2 \underbrace{U_1^{-1}}_{\text{u.t.}}}_{\text{u.t.}}$$

l.t.

1's on diagonal

So $L_2^{-1} L_1 = U_2 U_1^{-1}$ is ut & lt

Hence diagonal, w/ 1's (only) on diagonal

The identity matrix

So $L_2^{-1} L_1 = I \rightarrow L_1 = L_2$

& $U_2 U_1^{-1} = I \rightarrow U_2 = U_1$

QR Decompositions

$A = [\underset{\sim}{A_1} | \underset{\sim}{A_2} | \cdots \underset{\sim}{A_n}]$ there exists unitary matrix Q

and an upper triangular matrix R so that $A = QR$

$$X \quad [\underset{\sim}{A_1} | \underset{\sim}{A_2} | \cdots \underset{\sim}{A_n}] R^{-1} = [\underset{\sim}{Q_1} | \underset{\sim}{Q_2} | \underset{\sim}{Q_3} | \cdots | \underset{\sim}{Q_m}] \begin{bmatrix} \times & \times & \times & \times & \times & \times \\ 0 & \times & \times & \times & \times & \times \\ 0 & 0 & \times & \times & \times & \times \\ & & \vdots & & & \end{bmatrix}$$

$m \times n \qquad m \times m \qquad m \times n$

$\underset{\sim}{A_1} = \times \underset{\sim}{Q_1}$

$\hat{A_2} = \times Q_1 + \times Q_2$

Chapter 1: Reflectors (Householder Matrix)