

Math 390

Friday, March 5

LU & QR Decompositions

Strassen Multiplication

$$P = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix}$$

$$T = \underline{ae} - dh$$

$$[P]_{11} = T - Q + R$$

$$\mathcal{O}(n^3) / \mathcal{O}(n^{2.7...})$$

Thm 2.1.1 LU Decomposition is unique

Suppose  $A = L_1 U_1 = L_2 U_2$  ( $L_1, L_2$  have 1's on diagonal)

$$L_2^{-1} L_1 = L_2^{-1} A A^{-1} L_1$$

l.t.

$$= L_2^{-1} (L_2 U_2) (L_1 U_1)^{-1} L_1$$

l.t.

$$= L_2^{-1} L_2 U_2 U_1^{-1} L_1^{-1} L_1$$

$$= U_2 \underbrace{U_1^{-1}}_{\text{l.t.}}$$

u.t.

So  $L_2^{-1} L_1 = U_2 U_1^{-1}$  is u.t. & l.t.

Hence diagonal, w/ 1's (only) on diagonal  
The identity matrix

$$\text{So } L_2^{-1} L_1 = I \rightarrow L_1 = L_2$$

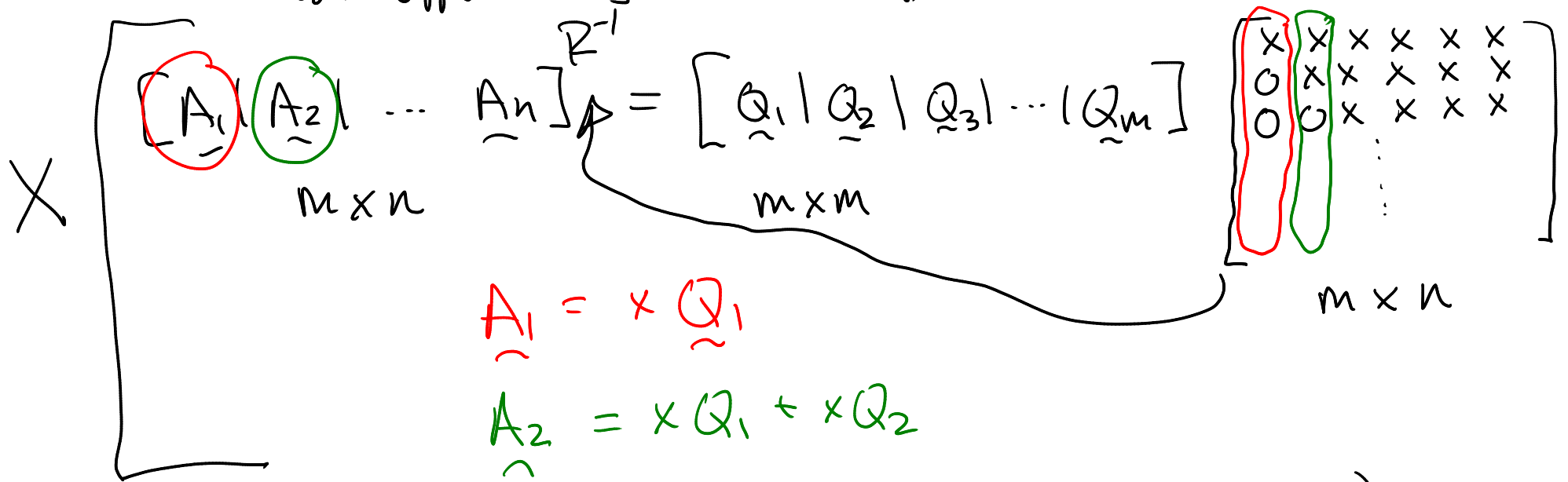
$$\neq U_2 U_1^{-1} = I \rightarrow U_2 = U_1$$

1's on diagonal

# QR Decompositions

$A = [\underline{A}_1 | \underline{A}_2 | \dots | \underline{A}_n]$  there exists unitary matrix  $Q$

and an upper triangular matrix  $R$  so that  $A = QR$



Chapter 1: Reflectors (Householder Matrix)