Math 390 Thursday, March 25 FCLA OD / Schur Decomposition

390/FCLA Theorem SUT

Section SD

Similar to Upper Triangular

matrix style

FCLA Theorem UTMR

Section OD

Same content as SUT

but in linear transformation style

Bonus: says that the eigenvalue count on the diagonal is the algebraic multiplicity

390/FCLA Theorem UTEC "Multiplicity of an Eigenvalue"

Section CP/ME

eigenvalue count matrix style

Fri - OD / Schur

Thu Fri Cholesky, plus

Project Proposals

April 5 (Monday) Problem Session

April 6 Exam 2

Transcript from Tuesday

Computation of Eigenvalues

Iterative techniques

$$A = \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix} \xrightarrow{\text{similarity transformations}} \begin{bmatrix} \lambda_1 & x & x & x \\ & \lambda_2 & x & x \\ & 0 & \ddots & \\ & & & \lambda_n \end{bmatrix}$$

$$S_1^{-1}(\ )S_1$$

$$S_2^{-1}S_1^{-1}(\ )S_1 S_2$$

similarity transformation to target one entry to become zero (or close!) while preserving zeros.

Impossible to do this exactly

Theorem OBUTR (FCLA OD)

Orthonormal Basis for Upper-Triangular Representation

matrix style A square matrix $\Rightarrow$ unitary U so that

$$\underbrace{U^* A U}_{\text{similarity}} = T \leftarrow \text{upper triangular}$$

Proof Use QR

There is S so that

$$S^{-1} A S = T$$

$$A = S T S^{-1}$$

S full rank, square

$$S = Q R$$

Q unitary

R upper triangular, diagonal 1's

$$= (QR) T (QR)^{-1}$$

$$= Q (R T R^{-1}) Q^*$$

all upper triangular, product upper-triangular

$$= Q \hat{T} Q^* \longrightarrow Q^* A Q = \hat{T}$$

Theorem OD $\quad A$ square

$$U^* A U = D \iff A \text{ normal}$$

diagonal
w/ eigenvalues