Graph Theory

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Contents

Chapter 1

Connectivity

1.1 Introduction

Cross-file mathjax reference: 2.5

We might rightly begin an exploration of graph theory with topics grouped together with a common theme of connectivity.

1.2 Walks, Paths, Trails

There is often considerable confusion about these terms. Consider the following definition.

Definition 1. *A **walk** in a graph is a sequence of vertices such that consecutive vertices in the sequence are adjacent in the graph.*

Theorem 1. *Suppose that A is the adjacency matrix of a graph, then the number of walks in the graph of length k, between vertices v_i and v_j is the (i, j) entry of A^k.*

Proof. Induction on k, with the definition of matrix multiplication.

□

So this definition does not preclude "visiting" a vertex more than once, nor going back-and-forth along a single edge.

Chapter 2

Algebraic Graph Theory

2.1 Introduction

This is one of my favorite topics.

2.2 Adjacency Matrices

2.2.1 Introduction

Might as well begin here.

We can have a variety of displayed equations, basically from the amsmath package. For example, a single displayed equation, with no number, and hence no referencing is possible:

$$x^2 + y^2 = 25$$

We can can have a single numbered equation, so referencing is possible:

$$x^4 + y^4 = 81 \tag{2.1}$$

Or several equations in a row, none of them numbered.

$$a^2 - b_3 + 6 = 123$$
$$a^2 - b_4 + 6 = 123$$
$$a^2 - b_7 + 6 = 123$$

Or several equations in a row, all of them numbered.

$$a^2 - b_3 + 6 = 123 \tag{2.2}$$
$$a^2 - b_4 + 6 = 123 \tag{2.3}$$
$$a^2 - b_7 + 6 = 123 \tag{2.4}$$

We can selectively not number equations in a group that are all numbered.

$$a^2 - b_3 + 6 = 123 \tag{2.5}$$
$$a^2 - b_4 + 6 = 123$$
$$a^2 - b_7 + 6 = 123 \tag{2.6}$$

We can selectively number equations in a group that are all unnumbered.

$$a^2 - b_3 + 6 = 123$$
$$a^2 - b_4 + 6 = 123 \tag{2.7}$$
$$a^2 - b_7 + 6 = 123$$

2.2.2 The Basics

We should first make a definition.

Definition 2. *Given a graph G, we define the* ***adjacency matrix*** $A(G)$ *as the* 0-1 *matrix given by:*

$$A(G) = \begin{cases} 0 & \text{if } v_i \text{ and } v_j \text{ are adjacent} \\ 1 & \text{otherwise} \end{cases} \tag{2.8}$$

When no confusion will result, we will denote this matrix simply as A.

2.3 Eigenvalues

This is where the fun would start. But instead we will practice referencing some displayed equations, such as the first numbered single display, Equation 2.1 and the selectively numbered equation 2.7.

Test refs: 2.5, 2.6

Chapter 3

Gallery

3.1 Introduction

One of the best things about graph theory is that you can draw pictures. Here is a classic.

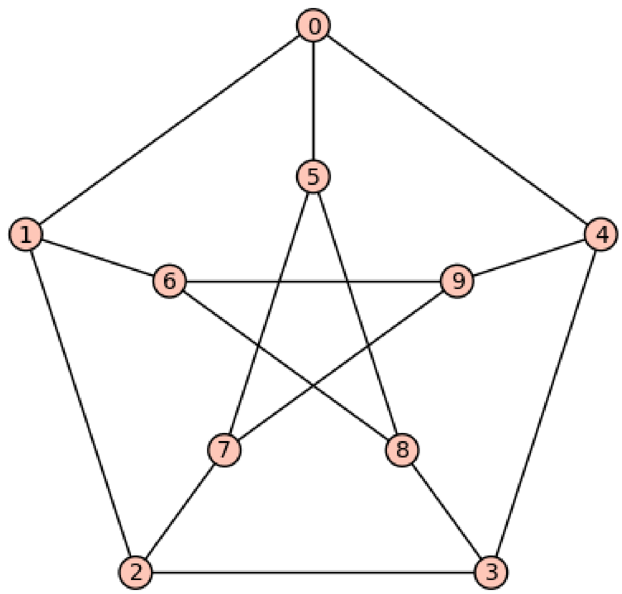

Figure 3.1: The Petersen graph

I also like Heawood's graph.

3.2 Regular Graphs

Regular graphs have a certain amount of combinatorial symmetry.

Definition 3. *We call a graph* ***regular*** *if every vertex has the same number of incident edges. This common number is called the* ***degree*** *of the graph.*

Very pretty, no?

Let's reference a previous definition on walks: Definition 1.

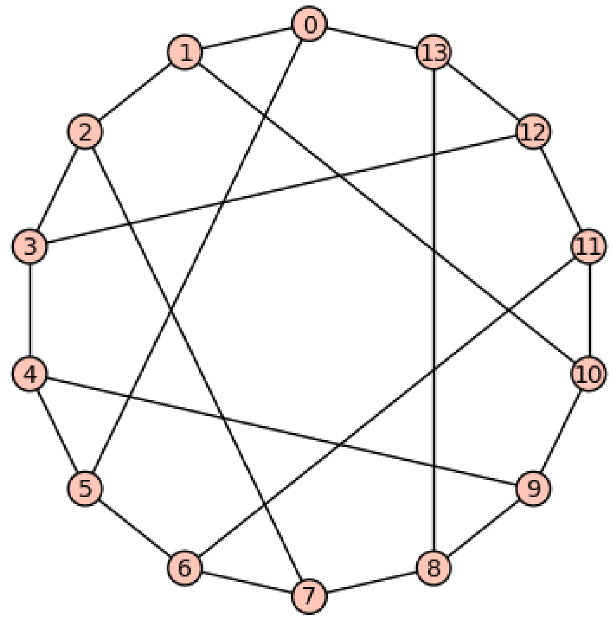

Figure 3.2: The Heawood graph

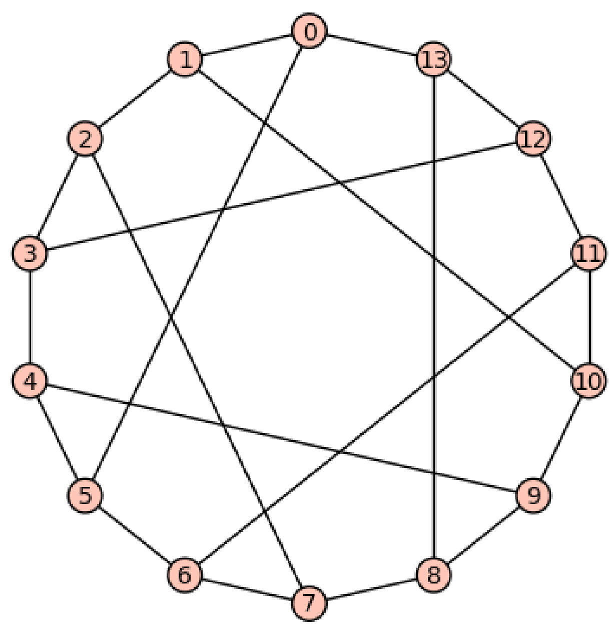

Figure 3.3: A regular graph

And a reference to the first Heawood graph: Figure 3.2.
There is a nice proof by induction at .