# Textbooks as Sage Notebooks

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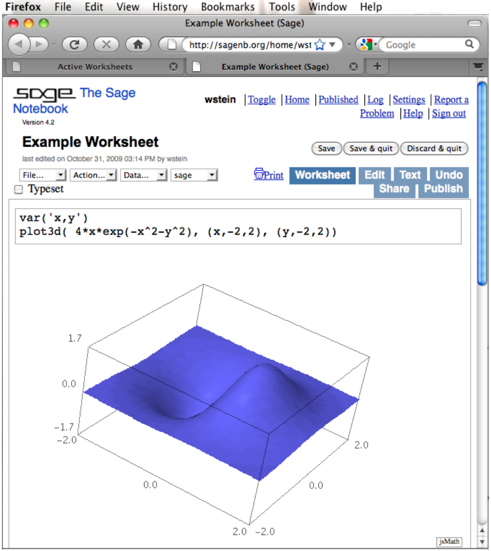
# Sage

- Mission: Creating a viable free open source alternative to Magma, Maple, Mathematica and Matlab.
- Unifies over 100 open source packages for mathematics and scientific computation.
  - R, Maxima, GAP/GP, Pari, Singular
  - LAPACK, FLINT, SciPy, NumPy
- ≈300,000 lines of new Python/Cython code
- User language is Python
- Use command-line or “Notebook” interface
- Notebook runs in any web browser
- Communicates with a server — local or remote
- Web Site: `sagemath.org`
- Public Notebook Server: `sagenb.org`

# Sage Notebook




Sage Notebook in Firefox Browser on OS X

# Sage Notebook

- Web 2.0 Application, AJAX
- Extensive use of Javascript

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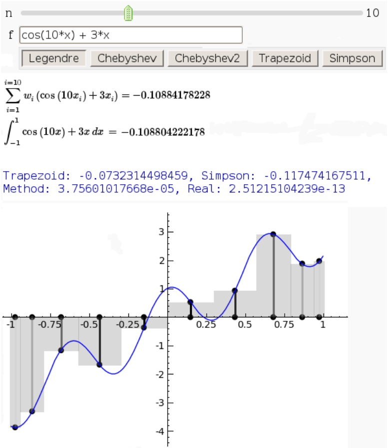
- Built-in: Tools to publish and share worksheets

# Sage "Interacts"

- Interactive demonstrations
- Like Java applets
- Like Mathematica's Manipulate
- Just a Python function
- Easily construct
  - Sliders
  - Checkboxes
  - Selection boxes
  - Input fields
  - ...
- Process with Sage, Python
- Output: 2D, 3D graphics
- Output: HTML, LaTeX
- Fast evolving feature of Sage

Gaussian Quadrature Interact
(by Jason Grout)



# Sage-Enhanced Textbooks

- Convert LaTeX to jsMath with tex4ht
- Easily modify this to be a Sage worksheet
- Incorporate:
  - Empty Sage input cells
  - Live, editable, Sage example code
  - One-click runnable interacts
- In their book, reader can:
  - Execute Sage commands
  - Experiment with Sage example code
  - Experiment with interacts
  - Copy and modify interacts
  - Program in Python
  - Annotate, including LaTeX

# Prototype Sage-Enhaced Textbook

An open source development mantra:

Release EARLY, release often.

- Hand-crafted prototype
- Sampling of a section on linear transformations
- All begins with LaTeX source
- Lots of cut/paste, can all be automated
- More notebook support coming

# PDF output



So by Definition LT [519], $P$ is a linear transformation. ⊠

So the multiplication of a vector by a matrix "transforms" the input vector into an output vector, possibly of a different size, by performing a linear combination. And this transformation happens in a "linear" fashion. This "functional" view of the matrix-vector product is the most important shift you can make right now in how you think about linear algebra. Here's the theorem, whose proof is very nearly an exact copy of the verification in the last example.

**Theorem MBLT**
**Matrices Build Linear Transformations**
Suppose that $A$ is an $m \times n$ matrix. Define a function $T\colon \mathbb{C}^n \to \mathbb{C}^m$ by $T(\mathbf{x}) = A\mathbf{x}$. Then $T$ is a linear transformation. □

**Proof**

$$\begin{aligned} T(\mathbf{x}+\mathbf{y}) &= A(\mathbf{x}+\mathbf{y}) && \text{Definition of } T \\ &= A\mathbf{x} + A\mathbf{y} && \text{Theorem MMDAA [231]} \\ &= T(\mathbf{x}) + T(\mathbf{y}) && \text{Definition of } T \end{aligned}$$

and

$$\begin{aligned} T(\alpha\mathbf{x}) &= A(\alpha\mathbf{x}) && \text{Definition of } T \\ &= \alpha(A\mathbf{x}) && \text{Theorem MMSMM [232]} \\ &= \alpha T(\mathbf{x}) && \text{Definition of } T \end{aligned}$$

So by Definition LT [519], $T$ is a linear transformation. ■

So Theorem MBLT [526] gives us a rapid way to construct linear transformations. Grab an $m \times n$ matrix $A$, define $T(\mathbf{x}) = A\mathbf{x}$ and Theorem MBLT [526] tells us that $T$ is a linear transformation from $\mathbb{C}^n$ to $\mathbb{C}^m$, without any further checking.

We can turn Theorem MBLT [526] around. You give me a linear transformation and I will give you a matrix.

# jsMath output

Section LT Linear Transformations - Mozilla Firefox

File Edit View History Bookmarks Tools Help

http://linear.ups.edu/jsmath/latest/fcla-jsmath-latestli51.html#x52-242000

Section LT Linear Transformations

So the multiplication of a vector by a matrix "transforms" the input vector into an output vector, possibly of a different size, by performing a linear combination. And this transformation happens in a "linear" fashion. This "functional" view of the matrix-vector product is the most important shift you can make right now in how you think about linear algebra. Here's the theorem, whose proof is very nearly an exact copy of the verification in the last example.

**Theorem MBLT**
**Matrices Build Linear Transformations**
Suppose that $A$ is an $m \times n$ matrix. Define a function $T : \mathbb{C}^n \rightarrow \mathbb{C}^m$ by $T(\mathbf{x}) = A\mathbf{x}$. Then $T$ is a linear transformation. □

**Proof**

| | | | |
|---|---|---|---|
| $T(\mathbf{x}+\mathbf{y})$ | $= A(\mathbf{x}+\mathbf{y})$ | Definition of $T$ | |
| | $= A\mathbf{x}+A\mathbf{y}$ | Theorem MMDAA | |
| | $= T(\mathbf{x})+T(\mathbf{y})$ | Definition of $T$ | and |
| $T(\alpha\mathbf{x})$ | $= A(\alpha\mathbf{x})$ | Definition of $T$ | |
| | $= \alpha(A\mathbf{x})$ | Theorem MMSMM | |
| | $= \alpha T(\mathbf{x})$ | Definition of $T$ | *l*.313 |

So by Definition LT, $T$ is a linear transformation. ■

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**Example MFLT**
**Matrix from a linear transformation**
Define the function $R : \mathbb{C}^3 \rightarrow \mathbb{C}^4$ by

jsMath

Find: Previous Next Highlight all Match case

# Sage Development

- Google Groups: sage-edu
- Use, test, comment to influence development
- Construct and share interacts: examples on Sage wiki
- Add new code to Sage for mathematics
- Develop new features for notebook using Javascript
- JOIN US!

Posted at `http://buzzard.ups.edu/talks.html`

# Sage Enhanced Textbook Prototype

Section RREF Reduced Row-Echelon Form - Mozilla Firefox

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http://linear.ups.edu/jsmath/latest/fcla-jsmath-latestli18.html#x19-340(

Google

To write the set of solution vectors in set notation, we have

$$S = \left\{ \begin{bmatrix} 3-x_3 \\ 2+x_3 \\ x_3 \end{bmatrix} \mid x_3 \in \mathbb{C} \right\}$$

We'll learn more in the next section about systems with infinitely many solutions and how to express their solution sets. Right now, you might look back at Example IS. ⊠

Generate new matrix

Operation: Automatic | Swap A and B | Multiply A | Multiply A & Add to B

Row A: 2 Row B: 3 Multiple: 4

$$\begin{pmatrix} 1 & 2 & -1 & -3 \\ 0 & -1 & 2 & 1 \\ 0 & 4 & -8 & -4 \end{pmatrix} \xrightarrow{4R_2+R_3 \to R_3} \begin{pmatrix} 1 & 2 & -1 & -3 \\ 0 & -1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

**Theorem RREFU**
**Reduced Row-Echelon Form is Unique**
Suppose that $A$ is an $m \times n$ matrix and that $B$ and $C$ are $m \times n$ matrices that are row-equivalent to $A$ and in reduced row-echelon form. Then $B = C$. □

**Proof** We need to begin with no assumptions about any relationships between $B$ and $C$, other than they are both in reduced row-echelon form, and they are both row-equivalent to $A$.