Publishing Mathematics with XML

Rob Beezer
beezer@pugetsound.edu

Department of Mathematics and Computer Science
University of Puget Sound

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Truths

- Much information/knowledge is discovered/learned from screens
- The Internet is a *publishing* platform
- MathJax makes math look good in a browser
- The Sage Cell and Sage Cloud are important developments
- Doctesting Sage examples is *critical*
- Browsers: yes! E-Books: not quite there yet.

The Problem with LaTeX

- It does not really separate content and presentation
- It is really, really hard to parse and convert
- It does not capture the structure of a document

XML - eXtensible Markup Language

- Hierachical tree-like structure imposed on text
- Powerful tools to edit, validate, parse, convert
- Minimal reserved characters (primarily <, &)
- HTML (XHTML) is an example
- "XML Application" - tags and converters
- Downside: verbose (harder to read than LaTeX?)

My Experiments

- FCLA converted Summer 2012
- Chris Godsil's "Explorations in Algebraic Graph Theory with Sage"
- Tom Judson's AATA Instructor Manual
- Exams, letters
- Classroom Note to submit to the Monthly

A Language for Mathematics

Properties:

- Structure of academic works (articles, books, chapters, sections)
- Support for mathematics (e.g. displayed mathematics)
- Sage code (static or dynamic)
- Usual: citations, cross-references, ToC, numbered equations

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Outputs:

- Web pages (MathJax, Sage Cell)
- Latex $\rightarrow$ PDF
- Worksheets, Notebooks (sagenb, Salvus/Cloud)
- Doctest file
- In-browser preview (CSS or XSLT stylesheet)
- E-Books

Current Thinking

- Structure: book, article, chapter, section, subsection
- Math: inline, displayed, aligned (with, without numbering)
- Sage: sage, input, output (random, not tested, etc)
- Recycle usual HTML: p, ol, li, em, q, etc.
- Borrow from DocBook: Figures, tables
- Borrow from DocBook: metadata, bibliography
- Keep It Super Simple, but grow "organically"

SHUTTLEWORTH
FUNDED

XML Source

New Open Back Forward Save Save As Close Undo Redo

Documents Projects Snippets

```
<title>Extended Echelon Form and Four Subspaces</title>
<author>Robert A. Beezer</author>
<email>beezer@pugetsound.edu</email>
<department>Department of Mathematics and Computer Science</department>
<institution>University of Puget Sound</institution>
<date>DRAFT June 14, 2013 DRAFT</date>

<p>It is well known that if <m>A</m> is a nonsingular matrix of size <m>n</m>, we can augment with the identity matrix of the same
size, bring the <m>n\times 2n</m> matrix to reduced row-echelon form, and then the final <m>n</m> columns will contain the inverse of
<m>A</m>.  What happens if <m>A</m> is singular, or perhaps not even square?</p>

<p>Given a matrix <m>A</m>, four associated subspaces are of special interest: the column space <m>\col{A}</m>, the (right) null space
<m>\nsp{A}=\{\vect{x}\,\vert\,A\vect{x}=\vect{0}\}</m>, the row space <m>\row{A}</m> and the left nullspace <m>\nsp{\transpose{A}}</m>.
Strang has dubbed these <q>The Four Fundamental Subspaces</q> (XX: citation) and their properties and relationships are indeed
fundamental, as well as being varied and interesting.  Bases for all four of these subspaces can be obtained easily from <m>C</m> and
<m>L</m>.  By-products are a canonical description of the row operations which bring a matrix to reduced row-echelon form, and the fact
that row rank and column rank are equal.  We only know of one other textbook, besides our own, which describes this procedure (XX:
citation, beezer, stuart find).  So the purpose of this note is to make this approach better known.</p>

<lemma>

<statement><p><m>\col{A}=\nsp{L}</m></p></statement>

<proof>
<p>Let <m>r</m> denote the number of rows in <m>C</m>.  Suppose that <m>\vect{y}\in\col{A}</m>.  Then there exists a vector
<m>\vect{x}</m> such that <m>A\vect{x}=\vect{y}</m>.  So
<dm>\twopartcol{C\vect{x}}{\vect{0}}
= \twopartcol{C\vect{x}}{0\vect{x}}
= B\vect{x}
= J\vect{y}
= \twopartcol{K\vect{y}}{L\vect{y}}</dm>
The last <m>m-r</m> entries of this vector equality express the condition that <m>\vect{y}\in\nsp{L}</m>.</p>

<p>Conversely, suppose <m>\vect{y}\in\nsp{L}</m>.  Consider <m>K\vect{y}</m>, and because <m>C</m> has full rank, there is a vector
<m>\vect{x}</m> such that <m>C\vect{x}=K\vect{y}</m>.  Then,
<dm>J\vect{y}
= \twopartcol{K\vect{y}}{L\vect{y}}
= \twopartcol{C\vect{x}}{\vect{0}}
= \twopartcol{C\vect{x}}{0\vect{x}}
= B\vect{x}</dm>
Thus, <m>A\vect{x}=\vect{y}</m> and <m>\vect{y}\in\col{A}</m>.</p>

</proof>
</lemma>
```

Line: 36 Col: 5 INS LINE UTF-8 extended-echelon-form.xml

Search and Replace Current Project

Demonstrations